Write your name here:

Math 109, Calculus II, spring term 2010, Prof. Andrew Salch.
Midterm Exam 1.

- Make sure to write your name on the top of this page.
- Show your work for full credit. You may get partial credit if you solve part of a problem correctly.
- You may not use any of the following things on this test: a calculator, a computer, a cell phone, an Ipod, anything else with an LCD screen, notes, books, people (other than yourself), alcohol, tobacco, firearms, explosives, a calculator, other students' tests, anything that has headphones, Morse code, two calculators, three calculators, or anything else that you have a guilty or rebellious feeling about using on this test.
- Do not be worried if you cannot answer every problem on the test; I do not expect that you will be able to answer all of the problems in the 50 minutes alloted for the exam. Do as much as you can, and you will receive partial credit for problems which are partially complete. There will be a curve on this exam.
- A few useful formulas:

$$\begin{aligned} \sin^2 x &= \frac{1-\cos(2x)}{2} \\ \cos^2 x &= \frac{1+\cos(2x)}{2} \\ \sec^2 x &= 1+\tan^2 x. \end{aligned}$$

For graders' use:

Problem number	Your score	Maximum score
1		20
2		20
3		20
4		20
5		20
6		20
7		20
Total		140

Problem 1. Compute the antiderivative

$$\int e^x \cos x \; dx.$$

Problem 2. Compute the antiderivative

$$\int \sin^2 x \cos^5 x \; dx.$$

Problem 3. Compute the antiderivative

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx,$$

where a is a constant real number. (Your answer will be in terms of a.)

Problem 4. Compute the antiderivative

$$\int \frac{x^3}{(x-1)^2}\,dx.$$

Problem 5. Solve the differential equation

$$\frac{dy}{dx} + y \ln x = \frac{1}{x^x},$$

where x is restricted to the positive real numbers (i.e., $x > 0$).

(Hint: $\frac{1}{x^x} = e^{-x \ln x}$.)

Problem 6. Compute the arclength of the parametric curve

$$\begin{aligned} x(t) &= at, \\ y(t) &= \frac{1}{3}t^3, \end{aligned}$$

from $t = 0$ to $t = 1$, where a is a constant real number. You answer will be in terms of a. (Hint: the formula

$$\int \sec\theta \; d\theta = \ln|\sec\theta + \tan\theta| + C$$

may be useful to you.)

Problem 7. Suppose a, b are constant real numbers. Find all real numbers θ such that the tangent line to the polar curve

$$r(\theta) = a + b\sin\theta$$

is horizontal at θ. (Your answer will be in terms of a and b.)