Write your name here:

Math 109, Calculus II, spring term 2010, Prof. Andrew Salch.
Final exam.

- Make sure to write your name on the top of this page.
- Show your work for full credit. You may get partial credit if you solve part of a problem correctly.
- You may not use any of the following things on this test: a calculator, a computer, a cell phone, an Ipod, anything else with an LCD screen, notes, books, people (other than yourself), alcohol, tobacco, firearms, explosives, a calculator, other students' tests, anything that has headphones, Morse code, two calculators, three calculators, or anything else that you have a guilty or rebellious feeling about using on this test.
- Do not be worried if you cannot answer every problem on the test; I do not expect that you will be able to answer all of the problems in the 50 minutes alloted for the exam. Do as much as you can, and you will receive partial credit for problems which are partially complete. There will be a curve on this exam.
- A few useful formulas:

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \cos^2 x = \frac{1+\cos(2x)}{2}$$
$$\sec^2 x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{x^2+1}.$$

For graders' use:

Problem number	Your score	Maximum score	Problem number	Your score	Maximum score
1		20	8		20
2		20	9		20
3		20	10		20
4		20	11		20
5		20	12		20
6		20	13		20
7		20	14		20
			Total		280

Problem 1. Compute the antiderivative

$$\int \sec^2(2x)\tan(2x)\ dx.$$

Problem 2. Compute the antiderivative

$$\int \frac{x-1}{x(x^2+1)}\, dx.$$

Problem 3. Compute a function $y(x)$, defined for all $x \in [0, \infty)$, such that $y(1) = 0$ and

$$x\frac{dy}{dx} - 4y = x^6 e^x.$$

Problem 4. Consider the polar curve defined by the equation

$$r(\theta) = \cos\theta - \sin\theta.$$

Compute all values of θ in the interval $[0, 2\pi]$ such that the tangent line (in the xy-plane) to this polar curve at θ has slope 1.

Problem 5. Consider the parametric curve in the xy-plane given by the equations

$$\begin{aligned} x(t) &= \cos t + t \sin t, \\ y(t) &= \sin t - t \cos t. \end{aligned}$$

Compute the arclength of this curve from $t = 0$ to $t = \pi$.

Problem 6. Find all positive real numbers t such that the series

$$\sum_{n=0}^{\infty} \left(\frac{tn+1}{2n+1} \right)^n$$

converges.

Problem 7. Prove that the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$$

is *conditionally* convergent. (So you need to show that it is convergent, and then you also need to show that it isn't absolutely convergent.)

Problem 8. Compute the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n2^n}.$$

Make sure you also determine whether or not the power series converges at the endpoints of the interval of convergence.

Problem 9. Suppose k is a real number. Compute the Taylor series for the function

$$f(x) = e^{kx}$$

centered at -1.

Problem 10. Compute the Maclaurin series for the function

$$f(x) = \frac{1}{(x+3)^2}.$$

(Hint: $\frac{d}{dx}\frac{1}{x+3} = \frac{-1}{(x+3)^2}$.)

Problem 11. Recall that the Maclaurin series for the arctangent function is

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1},$$

and $\arctan 1 = \frac{\pi}{4}$. Use Taylor's inequality to estimate $\frac{\pi}{4}$ to an error of within $\frac{1}{8!}$.

(Hint: if $n \geq 0$ and $f(x) = \arctan x$, then

$$|f^{(n+1)}(x)| \leq 1$$

for all $x \in [-1, 1]$.)

Problem 12. Compute the improper integral

$$\int_{-\infty}^{\infty} \frac{1}{x^2+1}\, dx.$$

Problem 13. Recall that the Maclaurin series for the arctangent function is

$$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1},$$

and the Maclaurin series for the exponential function e^x is

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

Compute the fifth Taylor polynomial of $e^x \arctan x$ centered at 0.

Problem 14. Use Taylor's inequality to prove that e^x is equal to its Maclaurin series

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

for every real number x.