HOMEWORK ASSIGNMENT

All the assignments are from the textbook, unless otherwise specified.

- Week 1.
  Section 1.1 : $\#1, 7, 17, 22, 23$
  Section 1.2 : $\#3, 5, 7, 17$
  Section 1.3 : $\#1-6, 11, 17$
  Section 2.1 : $\#9, 13, 30, 33$

  Due date: February 1/2nd.

- Week 2.
  Section 2.2 : $\#1, 3, 17, 21, 23$
  Section 2.3 : $\#13, 16$
  Section 2.4 : $\#1, 2, 4, 9, 13, 25, 32$

  Due date: February 7th.

- Week 3.
  Section 2.5 : $\#3, 7, 10, 11, 14, 20, 27$
  Section 2.6 : $\#3, 7, 13, 18$ and the following Fish Pop. Prob.

  Extra Bifurcation Problem:

  Consider the population model for a species of fish in a lake

  $$\frac{dP}{dt} = -\frac{P^2}{50} + 2P,$$

  where P is measured in thousands of fish and t is measured in years. The US Fish and Wildlife Service, which is managing the lake, wants to issue fishing licenses for the harvesting of the fish (this amounts to a constant term being subtracted off of the right hand side above, which is a function of h, the number of licenses issued). Each fishing license is valid for the annual take of 3000 fish. Draw a bifurcation diagram for the above ODE with the added parameter part, and answer the following questions.

  (a) What is the largest number of licenses that can be issued if the goal is to keep a stable population of fish in the lake over the long term?

  (b) If the largest number of licenses is actually issued, what is the expected long term stable population of fish in the lake?

(c) As a consultant to the USFWS, discuss the ramifications of issuing the maximal number of licenses allowed by a mathematical model in the presence of real world issues which may temporary affect populations (drought, flooding, unlawful fishing, pollution, etc.)

(d) Solve the IVP given by the above differential equation and the initial value $P(0) = 2$ (this corresponds to an initial population of 2000 fish in the lake, and an assumption that there will be no harvesting, $h = 0$).

Due date: February 14th.

- WEEK 4.
  Section 3.1 : $\#4, 16, 20, 21$
  Section 3.2 : $\#5, 9, 11, 14, 22$
  Section 3.3 : $\#1, 16, 18, 21$
  Section 3.4 : $\#10, 19, 24$

  Due date: February 21st.

- WEEK 5.
  Section 3.5 : $\#10, 12, 18, 20, 28$
  Section 3.6 : $\#3, 27, 29$
  Section 3.7 : $\#1, 13, 28, 30$ (also Resonance Prob.)

Extra 2nd Order ODE problem: Resonance

A car supported by a MacPherson strut (shock absorber system) travels on a bumpy road at a constant velocity v. The equation modeling the motion of the car is

$$80\ddot{x} + 10000x = 2500 \cos\left(\frac{\pi v t}{6}\right),$$

where x represents the vertical position of the cars axle relative to its equilibrium position, and the basic units of measurement are feet and feet per second. The constant numbers above are related to the characteristics of the car and the strut. Note that the coefficient of time t (inside the cosine) in the forcing term on the right hand side is a frequency, which in this case is directly proportional to the velocity v.

(a) Find the general solution to this non-homogeneous ODE. Note that your answer will have a term in it which is a function of v.

(b) Determine the value of v for which the solution is undefined (you should present your final answer in miles per hour, as opposed to feet per second).

(c) For a set of initial values $x(0) = 0, \dot{x}(0) = 0$, graph the solutions for a few values of v near your answer in part b and not so near. Discuss the differences in these graphs and the importance of the special value of v in part b. (Hint: This special value of v induces what is called resonance in the car).

Due date: February 28th.

- Week 6.
  Section 7.1 : $\#3, 5, 7, 13, 15$

  Due date: March 7th.

- Week 7.
  Section 7.3 : $\#4, 7, 12, 16$
  Section 7.4 : $\#4, 6$
  Section 7.5 : $\#15, 17, 24, 29, 31$

  Due date: March 21st.

- Week 8.
  Section 7.6 : $\#3, 5, 10, 17, 23$
  Section 7.7 : $\#1$ (added problem)
  Section 7.8 : $\#2, 7, 16$

Extra problem.
For the 1st order system of differential equations

$$x' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} x,$$

do the following:

(a) Find and state the general solution.

(b) Write out the fundamental matrix $\Psi(t)$ corresponding to this general solution.

(c) Find the particular solutions to the IVPs whose ODE is as above, and whose initial values are

(1) $x(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

(2) $x(0) = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

(3) $x(0) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

Write these out using $\Psi(t)$.

(d) Solve the IVP with the above ODE but with the initial values

$$x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(Hint: Best to use the general solution in part (a) for this.) These two solutions form the columns of a different fundamental matrix, called $\Phi(t)$. Do this and verify that

$$\phi(0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(e) Now again write out the particular solutions to the IVPs given in part (c), using $\Phi(t)$.

Due date: March 28th.

- Week 9.
  Section 9.1 : $\#3, 6, 15, 17, 20, 21$
  Section 9.2 : $\#5, 9, 14, 19, 21$ (use JODE)
  Section 9.3 : added problem

Extra Fixed Point Problem.

Problems 15 through 22 of Section 9.2 in the book are special. Given the system

$$\dot{x} = F(x, y)$$
$$\dot{y} = G(x, y)$$

one can find a solution of the associated ODE formed by taking the ratio of the derivatives of y and x,

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dx} = \frac{G(x, y)}{F(x, y)}$$

by finding a function $H(x, y)$ whose level sets wind up being the trajectories of the original system (many times this is because this last ODE is exact or separable).

Discuss why, in this case, the original system cannot have a sink or source as a fixed point.

Due date: April 4th.

- Week 10.
  Section 9.3 : $\#3, 7, 12, 17, 21$
  Section 9.4 : $\#4$
  Section 9.5 : $\#2$
  Section 9.7 : $\#3, 6, 16$

  Due date: April 11th.

- Week 11.
  Section 6.1 : $\#7, 16, 22$
  Section 6.2 : $\#6, 7, 8, 16, 19$

  Due date: April 18th.

- Week 12.
  Section 8.1 : $\#3a, 18$
  Section 8.2 : $\#3a, 14, 25$
  Section 8.3 : Find exact solution to 8.1.3 and compare to 8.1.3a, 8.2.3a, and 25

  Due date: April 25th.