The midterm will be composed by 4 problems. Each problem is worth 25 points. Here is a sample midterm.

(1) Show whether the following sequences are Cauchy.

(a)
$$x_n = \sum_{i=1}^{n} \frac{1}{i}$$

(b)
$$y_n = \sum_{i=1}^{n} 2^{-i}$$

Proof. The first sequence is not Cauchy. Indeed
$$x_{n^2} - x_n = \sum_{i=n+1}^{n^2} \frac{1}{i} \geq (n^2 - n - 1)\,\frac{1}{n^2}$$
which does not go to zero as $n \to \infty$.

The second sequence is Cauchy. Indeed it is a monotone sequence since $y_{n+1} - y_n = 2^{-n-1} > 0$, and it is bounded from above by 1. Hence, it converges to the limsup. □

(2) Using the Intermediate Value Theorem, show that a polynomial of odd degree has at least one real root.

Proof. See page 131 in the textbook. □

(3) Let A be a set. Show that if x is a limit point of A, then there exists a sequence x_n of distinct points in A which converge to x.

Proof. Suppose that x is a L.P. of A. By definition, the nbhd
$$(x - 1, x + 1)$$
contains infinitely many points of A and, in particular, a point $x_1 \neq x$ belonging to A. Likewise, the nbhd
$$(x - |x - x_1|/2, x + |x - x_1|/2)$$
contains a point $x_2 \neq x$ belonging to A. Notice that $x_2 \neq x_1$! Now just continue this to get the sequence $x_n \to x$.

(4) For the 4th problem you will have 5 True/False questions.