Hw Solutions:

9.1.1: a) F b) T c) F d) F e) T

9.2.10:

$$\begin{vmatrix} i & 2+i & 0 \\ -1 & 3 & 2i \\ 0 & -1 & 1-i \end{vmatrix} = 1\begin{vmatrix} 2+i & 0 \\ -1 & 1-i \end{vmatrix} + 3\begin{vmatrix} i & 0 \\ 0 & (1-i) \end{vmatrix}$$

$$-2i\begin{vmatrix} i & 2+i \\ 0 & -1 \end{vmatrix} = (2+i)(1-i) + 3(i)(1-i)$$

$$-2i(-i) = 4+2i$$

9.3.4: Write

$$A = \begin{pmatrix} 2 & 1 & -3 \\ 1 & -2 & 1 \\ 3 & 4 & -2 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

Then, Cramer's rule gives

$$x_1 = \begin{vmatrix} 1 & 1 & -3 \\ 0 & -2 & 1 \\ -5 & 4 & -2 \end{vmatrix} \Big/ \det(A)$$

$$\det(A) = 2\begin{vmatrix} -2 & 1 \\ 4 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$$

$$= 2\cdot 0 + 5 - 3\cdot 10 = -25$$

$$\det\begin{vmatrix} 1 & 1 & -3 \\ 0 & -2 & 1 \\ -5 & 4 & -2 \end{vmatrix} =$$

$$-2\begin{vmatrix} 1 & -3 \\ -5 & -2 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ -5 & 4 \end{vmatrix} = -\cancel{2\ldots} + 34 - 9$$

$$= 25$$

So $x_1 = -1$

$$\det\begin{vmatrix} 2 & 1 & -3 \\ 1 & 0 & 1 \\ 3 & -5 & -2 \end{vmatrix} =$$

$$-\begin{vmatrix} 1 & -3 \\ -5 & -2 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}$$

$$= 17 + 13 = 30$$

Thus $x_2 = 30/-25$

$$\det \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & 0 \\ 3 & 4 & -5 \end{vmatrix} = -\begin{vmatrix} 1 & 1 \\ 4 & -5 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & -5 \end{vmatrix}$$

$$= +9 + 26 = 35$$

$$x_3 = 35/-25$$

4.5.15.

Obviously If $\delta\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = aA_{11}A_{22} + bA_{11}A_{21} + cA_{12}A_{22} + dA_{12}A_{21}$

Then δ is 2-linear

Conversely, supposing δ is 2-linear, we get

$$\delta\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \delta\begin{pmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{pmatrix} + \delta\begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix}$$

$$= \delta\begin{pmatrix} 0 & A_{12} \\ 0 & A_{22} \end{pmatrix} + \delta\begin{pmatrix} 0 & A_{12} \\ A_{21} & 0 \end{pmatrix} + \delta\begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix} + \delta\begin{pmatrix} A_{11} & 0 \\ A_{21} & 0 \end{pmatrix}$$

$$= A_{12}A_{22}\,\overset{=a}{\delta\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}} + A_{12}A_{21}\,\overset{b}{\overset{\shortparallel}{\delta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}} + A_{11}A_{22}\,\overset{=c}{\delta\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}} + A_{11}A_{21}\,\overset{d=}{\delta\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

Done

(Thm 3.5, Sec. 3.2)

4.3.19: β a basis $\Leftrightarrow$ $Rank(\beta) = n$ $\Leftrightarrow$ β is Invertible $\Leftrightarrow$ $\det(\beta) \neq 0$