5.1.6: λ an eigenvalue for $T \Rightarrow \exists$ v s.t.

$$Tv = \lambda v$$

Then

$$[T]_\beta [v_\beta] = (Tv)_\beta = [\lambda v]_\beta = \lambda (v)_\beta$$

5.1.1 a) F b) T c) F d) F e) F
f) F g) F h) T i) T j) F k) F

5.2.19b:

$$\begin{bmatrix} 8-\lambda & 10 \\ -5 & -7-\lambda \end{bmatrix} = (8-\lambda)(\lambda+7) + 50$$

$$= \lambda^2 - \lambda - 6 \qquad \lambda = -2, 3$$

$\lambda = -2$:

$$\begin{bmatrix} 10 & 10 \\ -5 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\lambda = 3$: $$\begin{bmatrix} 5 & 10 \\ -5 & -10 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$$X(t) = \begin{pmatrix} c_1 e^{-2t} + 2c_2 e^{3t} \\ -c_1 e^{-2t} - c_2 e^{3t} \end{pmatrix}$$

5.2.7:

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

The

$$(\lambda - 1)(\lambda - 3) - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$\lambda = 5, -1$$

So

$$\lambda = 5 \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\lambda = -1 \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

The

$$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & -1/3 \end{pmatrix}$$

$$\Rightarrow A^n = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5^n & 0 \\ 0 & (-1)^n \end{pmatrix} \begin{pmatrix} 1/3 & 2/3 \\ 1/3 & -1/3 \end{pmatrix}$$