5.4.1 a) F b) T c) F d) F e) T f) T

g) T

5.4.18

a) Note $f(t) = \det(A - tI)$ so

$$f(0) = 0 \iff \det(A) = 0 \iff a_0 = 0$$

b) This is just a direct computation using the Cayley – Hamilton Theorem

7.3.5: Let $f(t) = t^3 - 2t^2 + t = t(t^2 - 2t + 1)$
$= t(t-1)^2$

Let $g(t)$ be the minimal polynomial of T. Then $\deg(g) \le 2$. Thm 7.12 gives

$$g \mid f$$

so

$$g = t, (t-1), t(t-1), (t-1)^2$$

$g = t \Rightarrow T = T_0$

$g = (t-1) \Rightarrow T = id$

$g = t(t-1) \Rightarrow T = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ ← In the right basis

$g = (t-1)^2 \Rightarrow$

$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ ← Again, in the right basis

But in this case T is not diagonalizable

7.3. Let $\mathfrak{F}_V, \mathfrak{F}_W$ denote the characteristic polynomials of T_V, T_W respectively.

Let p_V, p_W denote the Minimal polynomials of T_V, T_W respectively.

Use the division algorithm for polynomials to write

$$p_V = \alpha p_W + \beta$$

with $\deg(\beta) < \deg(p_W)$. We have

$\beta(T_W) = p_V(T_W) - \alpha p_W(T_W) = 0$

$\Rightarrow \beta = 0$, since p_W is the Minimal polynomial for T_W