Math 301: Introduction to Proofs
Problem Set 1
due: September 11, 2019

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Exercise 1. Analyze the logical forms of the following statements:

(i) If you walk outside in the rain and you don't carry an umbrella you'll get wet.
(ii) We'll have either a reading assignment or homework problems, but we won't have both homework problems and a test.
(iii) Either Emily and David are both telling the truth, or neither of them is.

Exercise 2. Analyze the logical forms of the following statements:

(i) Alice and Bob are not both in the room.
(ii) Alice and Bob are both not in the room.
(iii) Either Alice or Bob is not in the room.
(iv) Neither Alice nor Bob is in the room.

Exercise 3. Identify the premises (the assumptions) and conclusions of the following deductive arguments and analyze their logical forms. Is the logical reasoning valid, following the rules of inference we've learned? Explain why or why not.

(i) Thelma and Louise won?t both win the pole vault. Louise will win either the pole vault or the hurdles. Thelma will win the hurdles. Therefore, Louise will win the pole vault.
(ii) The main course will be either beef or fish. The vegetable will be either peas or corn. We will not have both fish as a main course and corn as a vegetable. Therefore, we will not have both beef as a main course and peas as a vegetable.
(iii) Either Emily or David is telling the truth. Either tslil or David is lying. Therefore, either Emily is telling the truth or tslil is lying.
(iv) Either sales will go up and the boss will be happy, or expenses will go up and the boss won?t be happy. Therefore, sales and expenses will not both go up.

Exercise 4. Let $x \in \mathbb{R}$. Prove that if x is irrational then $-x$ and $\frac{1}{x}$ are irrational.

Exercise 5. Prove that there is no smallest positive real number. That is prove that there is no positive real number a so that $a \leq b$ for all positive real numbers b.

Exercise 6. Let $a, b, c \in \mathbb{C}$, with $a \neq 0$, and consider the polynomial function $p(x) = ax^2 + bx + c$. Prove[1] that a complex number z is a root[2] of $p(x)$ if and only if

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Exercise 7. Let $a, b \in \mathbb{C}$ and let $p(x) = x^2 + ax + b$. The complex number $\Delta = a^2 - 4b$ is called the **discriminant** of the polynomial $p(x)$.

(i) Prove that $p(x)$ has two complex roots if $\Delta \neq 0$ and one complex root if $\Delta = 0$.
(ii) Assume $a, b \in \mathbb{R}$. Prove that $p(x)$ has no real roots if $\Delta < 0$, one real root if $\Delta = 0$, and two real roots if $\Delta > 0$.

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[1]This problem isn't intended as a test of your ability to complete the square. If your algebra is a little rusty feel free to look up the quadratic formula on Wikipedia. The point is to write a valid logical argument that establishes both implications expressed by this biconditional statement.
[2]A complex number z is a **root** of $p(x)$ iff $p(z) = 0$.