defn. A function \( f: A \rightarrow B \) is given by specifying for each \( a \in A \) a unique element \( f(a) \in B \).

**Exercise 1.** For any functions \( f: A \rightarrow B \) and \( g: B \rightarrow C \) define the \textbf{composite} function \( g \circ f: A \rightarrow C \) by specifying its action on elements.

defn. A function \( f: A \rightarrow B \) is
- \textbf{injective} iff \( f(a) = f(a') \) implies that \( a = a' \);
- \textbf{surjective} iff for every \( b \in B \) there exists \( a \in A \) so that \( f(a) = b \);
- \textbf{bijective} iff it is both injective and surjective.

**Exercise 2.** How many functions are there from a set of \( n \) elements to itself, where \( n \in \mathbb{N} \)? How many bijections are there between a set with \( n \) elements and itself?

**Exercise 3.** Assume \( A, B \neq \emptyset \). Prove that \( f: A \rightarrow B \) is an injective if and only if it has a \textbf{left inverse}: a function \( g: B \rightarrow A \) so that \( g \circ f = \text{id}_A \).

**Exercise 4.** Assume \( A, B \neq \emptyset \). Prove that \( f: A \rightarrow B \) is a surjective if and only if it has a \textbf{right inverse}: a function \( g: B \rightarrow A \) so that \( f \circ g = \text{id}_B \).

**Exercise 5.** Let \( f: A \rightarrow B \) be a function that has a left inverse \( g: B \rightarrow A \) and also a right inverse \( h: B \rightarrow A \). Prove that \( h = g \).

**Exercise 6.** Prove that function \( f: A \rightarrow B \) is a bijection if it has a \textbf{two-sided inverse}: a function \( g: B \rightarrow A \) so that \( g \circ f = \text{id}_A \) and \( f \circ g = \text{id}_B \). The data of \( A \), \( B \), \( f \), and \( g \) define a \textbf{isomorphism}.

**Exercise 7.** Two sets are \textbf{isomorphic} if there exists a bijection between them. Explain in your own words why all sets with three elements are isomorphic.

**Exercise 8.** Find a formula for the sum \( 1 + 3 + 5 + \cdots + (2n-1) \) of the first \( n+1 \) odd numbers (for \( n \) a positive integer) and prove (by induction) that your formula is correct.\(^1\)

\(^1\)For any set \( A \), the \textbf{identity function} \( \text{id}_A: A \rightarrow A \) is defined by \( \text{id}_A(a) = a \).

\(^2\)Hint: To discover the formula, first compute this sum for some particular small values of \( n \) and look for a pattern.