Math 401: Introduction to Abstract Algebra
Problem Set 9
due: April 15, 2019

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Read. §12, §14

Exercise 1. Find an example of a group $G$ with a subgroup $H$ so that
$$\{(x, y) \mid xy \in H\}$$
is not an equivalence relation on $G$.

Exercise 2. Find an example of a group $G$ with a subgroup $H$ so that
$$\{(x, y) \mid xyx^{-1}y^{-1} \in H\}$$is not an equivalence relation on $G$.

Exercise 3. For $G = A_4$ work out the left and right cosets of
(i) the subgroup $H = \{e, (12)(34), (13)(24), (14)(23)\}$ and
(ii) the subgroup $K = \{e, (123), (132)\}$
Verify that
(i) the left and right cosets coincide in the first case: $gH = Hg$ for all $g \in G$
(ii) but not in the second: for some $g \in G$, $gK \neq Kg$.
Challenge (optional): can you figure out what property holds of $H$ but not of $K$
that explains this?

Exercise 4. Let $G$ be a finite group and let $H$ be a subgroup which contains
precisely half of the elements of $G$. Show that $gH = Hg$ for every $g \in G$.

Exercises. §12 | 12.6, 12.9

Exercise 5*. Convince yourself that the braid group $B_3$ is infinite, non-abelian,
and is generated by the two braids $b_1$ and $b_2$ shown in Figure 12.4 on page 65.