Math 411: Honors Algebra I
Problem Set 9
due: November 15, 2017

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Exercise 1. Prove that if $0 = 1$ in a ring then the ring is the zero ring.

Exercise 2. A ring $R$ is **Boolean** if $a^2 = a$ for every $a \in R$. For any set $X$ prove that the set of subsets of $X$ becomes a Boolean ring with

\[
A + B := A \cup B - A \cap B \quad \text{the symmetric difference}
\]

\[
A \cdot B := A \cap B \quad \text{the intersection}
\]

by verifying the ring axioms.

Exercise 3. Prove or find a counter-example. If $R$ is a ring and $a, b \in R$ are zero divisors then $a + b$ is a zero divisor.

Exercise 4. Construct a field with 4 elements. The underlying abelian group is $\mathbb{Z}/2 \times \mathbb{Z}/2$ with $(0, 0)$ as the zero element and $(1, 0)$ as the multiplicative identity. The question is to define the multiplication table so that you get a field and not just a ring.

Exercise 5. Let $R$ be a commutative integral domain and consider the polynomial ring $R[x]$. Prove that the only units in $R[x]$ are the constant polynomials $f(x) = a_0$ where $a_0$ is a unit in $R$ and explain what goes wrong in your proof if $R$ is not an integral domain.

Exercise 6. Let $R$ be a commutative ring and consider the ring of power series $R[[x]]$. Prove that $1 - x$ is a unit in $R[[x]]$ by computing its inverse.\(^1\)

\(^1\)More generally, a power series $a_0 + a_1 x + a_2 x^2 + \cdots$ is a unit in $R[x]$ if and only if $a_0$ is a unit in $R$. 

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