MATHEMATICS, LIVE!

An Interview with Emily Riehl

Interviewer: Beth Malmskog, Colorado College

Emily Riehl is an incredibly accomplished early career mathematician, working at the interface of category theory and homotopy theory. She is also a stunning number of other things, including creative interdisciplinary scholar, working musician, and high-level athlete. A brief career outline: she did her undergraduate work at Harvard University, graduate work at Cambridge and the University of Chicago, was an NSF and Benjamin Peirce Postdoctoral Fellow at Harvard from 2011–2015, and is currently an assistant professor at Johns Hopkins University. Emily has been awarded an NSF standard grant and a CAREER award to support her work. She is the author of 21 published research articles, two books (*Categorical Homotopy Theory* and *Category Theory in Context*), and many other expository works. All this, and she also performs as a rock/alternative bass player and plays on the US women's national Australian Rules football team. The following interview is a compilation of email and Skype conversations from August 2017, while Emily was in Australia to compete in the AFL International Cup.

Q: Where did you grow up? How did you get started in math? Did you do a lot of math programs as a kid/teenager?

ER: I went to high school in a town called Normal, Illinois, about 45 minutes from UIUC. I always liked math, as far back as I can remember: looking for patterns in the calendar and so forth. I had several excellent teachers who encouraged me to double up on math courses and gave me activities to do outside the classroom—for instance a worksheet on base-four arithmetic completed in the hallway during late elementary school. I went to the Hampshire College Summer Studies in Mathematics program the summer before my junior year of high school. That was really my first acquaintance with proofs, and that's of course where I fell in love with mathematics. As a rising senior I spent the summer thinking about combinatorial group theory at the Research Science Institute at MIT, and once I figured out that that was a thing I enjoyed, I knew that this would be my career path.

This gave a lot of clarity to being an undergraduate; I went to Harvard and figured their standard math major courses would be sufficient to prepare me for graduate school, so in the meanwhile I got everything else out of my system. I took a year-long music theory course that was amazing and a couple courses in queer theory. The hardest course I took was a semester in American Intellectual History, which completely changed my reading practice, including my mathematical reading practice. I did all the reading, I attended every lecture, and then I took the midterm and got a C+. I was like, well, this isn't good, I must not be reading in a sufficiently engaged manner. Part of the issue was I have this fetishization of the book object—my books had to be pristine. After reading them, the spines would barely be cracked, and I certainly wouldn't write in them. So I went back and reread everything we had covered up to that point, and started underlining and writing marginal notes. Now, that's how I read all of my math papers, and it's a much better mode of engagement. And as a bonus it helps me discover when I've read something already. I still only use pencil in my books, though, and highlighters make me nervous.…

Q: How and why did you get into category theory? Is there a basic result that you can share that gives the flavor of what you love about it?

ER: For graduate school, I deferred from the University of Chicago for a year to go to Cambridge and do what they call a Part III. One of the courses they offered at Cambridge was in category theory, and I liked it instantly; I fell in love. I feel like it chose me as much as I chose it. And it was for the reason that I think that everyone chooses their field, ultimately: the proofs felt like the right way of thinking about mathematics. I felt right away that this is the sort of argument that I wanted to delve into.

Category theory can sound intimidating because it's highly abstract, but it's actually not that hard. Several of the most important definitions are quite elementary, and you can start stating and proving the theorems pretty quickly. Indeed, there's a common belief in category theory that once you understand the statement of the theorem, you can probably supply the proof yourself. Identifying the correct definitions is really the harder thing. The only reason that you typically don't learn category theory until graduate school is that it requires a rather high degree of mathematical sophistication to appreciate what it's for.

One of my favorite theorems in category theory is that right adjoints preserve limits—or, since you always get a dual theorem in category theory by simply "turning all the arrows around"—that left adjoints preserve colimits. This result specializes to explain why tensor products distribute over direct sums, why inverse images preserve intersections and unions while direct images only preserve unions, why quotients of topological spaces are formed by first identifying the appropriate points and then topologizing this quotient set. It's not so much that I appreciate having one proof instead of having to repeat the argument in each context but I feel that the category theoretic proof—which uses the fact that limits are characterized by a "mapping in" universal property, while colimits are characterized by a "mapping out" universal property—is the right one.

Q: How would you describe your research for this audience? Do you have a favorite result or idea from your research that you could briefly share?

ER: A long-time collaborator Dominic Verity and I are working to redevelop the foundations of $(\infty,1)$-category theory—or ∞-category theory to use the nickname given to them by Jacob Lurie. These sorts of weak infinite dimensional categories rear their heads in mathematical contexts where there are objects (for instance chain complexes, or points in some moduli space) that come with some notion of morphisms in each dimension (chain maps of varying degrees, higher homotopies). The foundations for this sort of category theory are brand new and rather daunting. Dom and I have introduced a new approach to proving the foundational theorems that is not reliant on a particular "model" of $(\infty,1)$-categories, such as the quasi-categories used by André Joyal and Lurie.

One of our papers introduces a new definition of a *homotopy coherent adjunction*. An adjunction is comprised of a pair of functors, the left and right adjoint referred to above, entangled by a natural duality. "Homotopy coherent" means roughly "infinite dimensional" and "free." The definition of a homotopy coherent adjunction is actually quite simple to state: it's essentially the same as the free 2-categorical adjunction, though this perspective doesn't explain why it ends up being homotopy coherent. To understand that, we introduce a graphical calculus in which the adjunction data is encoded by a strictly undulating squiggle crossing a finite number of parallel lines.

Q: You are early in your career, but you have written many, many papers, two books, and a lot of shorter expository work (like posts on The n-Category Café). How do you do so much stuff? Do you have any insights into how/why you are so productive?

ER: I read Hardy's *A Mathematician's Apology* in high school and my main takeaway was from the foreword written by C.P. Snow, who described Hardy's typical day: he devoted four hours in the morning, from 8–12, doing math, and then spent the afternoon watching cricket. It struck me as a particularly aspirational life style and so I've always focused more on working well than on working long hours. My main time-management strategy is to start work on the thing that is due the soonest last, when I'll be the most focused. So, for example, if I have a referee's report due in three months, I wait until almost three months have passed and then start to read the paper. I also do the preparation for my teaching in the hour or hour and a half before class, in what often feels like a race to figure out how to prove all the theorems before I rush across campus. Occasionally this gets me into trouble, for instance when I was trying set up a

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transfinite induction over the reals and couldn't understand why the intermediate stages were all "countable" (aside: I'm now firmly in the camp that believes that the axiom of choice is clearly true, while the well-ordering principle is clearly false). But this approach is very effective at reserving time for research and other long-term projects.

Q: How did you come to write your books, *Categorical Homotopy Theory* and *Category Theory in Context*, so early in your career? Was that a daunting undertaking?

ER: I view both books as political projects related to my somewhat unorthodox mathematical point of view. I didn't start out intending to write my first book, *Categorical Homotopy Theory*. Postdocs at Harvard are typically given an opportunity to teach a topics course during their first year as a way of introducing themselves to the department, and I wanted to use mine to make the case for the categorical point of view on certain topics in homotopy theory: homotopy limits and colimits, model categories, and $(\infty,1)$-categories.

To make my case, I knew I had to be really well prepared and I also wanted the audience to be able to catch up if they had to miss a lecture here or there; for instance, a few attendees were fellow postdocs who traveled frequently. So I decided to write lecture notes and by the end of the semester they came to almost 200 pages. These notes became the book.

The second book, which was planned, was also politically motivated in this sense. In my final semester at Harvard, I taught an undergraduate topics course entitled Category Theory in Context, during which I wrote the first draft of the book by that same name. It's written for mathematicians in other fields, though I hope discursive enough to be accessible to early graduate students or advanced undergraduates. Its aim was to provide a first introduction to the basic concepts of category theory, while simultaneously discussing the implications of these ideas in a wide variety of areas of mathematics on which category theory sheds light.

What distinguishes this manuscript from other introductory category theory texts is the amount of space I devote to examples. For instance, the chapter on "Universal properties, representability, and the Yoneda lemma" begins by listing 21 examples of representable functors and ends with a construction of the category associated to a set-valued functor in which any representing object is either initial or terminal. The preface to this chapter considers the functor that takes a graph to its set of n-vertex colorings, which is represented by the complete graph on n vertices, all colored distinctly.

This particular representable functor is one I dreamt up myself, but many of the other contextual examples were crowd-sourced. In the months before the class started, I put out a call online for illustrative applications of categorical ideas and received a number of excellent suggestions in response; the acknowledgments for this book run a page and a half. In addition, because the draft manuscript was always freely available online, I had a handful of early readers email me with detailed lists of suggestions and corrections; the same thing happened for *Categorical Homotopy Theory* as well. This feedback was fantastic, of course. I also imagine that the fact that the manuscript was already available on the web helped convince the publishers to allow me to continue to host a free PDF copy online.

I certainly would have been daunted by the prospect of writing a book were it not for the fact that with the first one I didn't realize until I was well underway that this was what I was doing. I also think it's easier psychologically to frame an expository project as "lecture notes" rather than a book. And, practically speaking, writing the first draft in installments to be posted on the internet twice a week after each class was very helpful in forcing me to keep on a tight schedule.

Q: What do you think are the best/worst parts of a life in math overall?

ER: The worst thing is how intellectually isolated we all are, how few people there are with whom we can share the insights that we find the most exciting, even among other mathematicians. For me personally I feel very frustrated that there is this huge part of my emotional life that most of the people whom I care about have no access to.

My favorite part of my job has always been giving talks. Research talks are my favorite, for the reasons alluded to above, but I also get some of that same thrill from giving colloquia or even from teaching. Even in high school, I enjoyed the performative aspects of lecturing. When I ran for student body president, my only real interest in the job was to give the campaign speech in front of the entire school.

Q: You begin your book *Categorical Homotopy Theory* with a quote from "On proof and progress in mathematics" by William Thurston: "... what we are doing is finding ways for people to understand and think about mathematics." How has Thurston's

perspective on mathematics as a community endeavor, with human understanding at its core, influenced your mathematical life?

ER: I've wondered at various points whether I should be concerned about the amount of time I end up devoting to expository projects, such as the books, because it does certainly eat into research time. This is one of many instances where I've found Thurston's essay, which I've re-read a few times now, to be helpful for keeping these kinds of projects in perspective. The passage you quote above is his definition of mathematical progress, which he sees as much broader than simply proving theorems. I happen to particularly enjoy mathematical exposition, so I think it makes sense—or as the economists would say, is a comparative advantage—for me to play that role in the broader community.

I read from a different section of this essay—on the difficulties of mathematical communication—at the introductory meeting for an AMS sponsored Mathematics Research Community workshop in Homotopy Type Theory that I co-organized this past June as a way of framing our goals for the week, which were largely to provide an opportunity for people who are not currently a part of that community (e.g., because they're doing their PhD at a place that doesn't have a faculty member working in that area) to find their way in.

Q: Tell me about your [former] band, Unstraight. Does the band still play since you left Boston? Do you play other music that I haven't managed to find on the internet?

ER: I played in Unstraight for 2–3 years during my postdoc at Harvard, but then they had to replace me when I moved to Baltimore. (They also had to be very accommodating with my crazy travel schedule; e.g., four months at MSRI, during which I flew back once so they wouldn't have to go that whole period without playing any shows.) I've filled back in for them twice since and hope to have more opportunities to do so in the future, but the hardest thing about being in a band is that the inflexibilities of the academic schedule mean that sometimes the job really has to come first.

On the flip side, the flexibilities of the academic schedule mean that sometimes weird things are possible. Last fall I played a couple of shows with Ami Dang (http://amidang.com) in a band she formed to play live versions of the songs off her recently released solo album *Uni Sun*. We're not on the record—available on Spotify; "Need to Fall" is my favorite track—but she wanted to mix it up for live shows. She booked a Wednesday night gig in Brooklyn and a Thursday night gig at her alma mater in Oberlin. I was picked up from campus after teaching my Wednesday multivariable calculus class, we drove the van to NYC. We played a show that evening, stayed overnight, then drove all day Thursday to Oberlin, played the show, and drove back overnight to get back in time for both of us to work on Friday. Of course the van broke down, but luckily it happened just as we were reaching the Baltimore city limits. I waited around for an hour for AAA but then Ami and I ended up catching a cab, which got me back to my office (unshowered, but oh well) in time to teach at 11 that Friday morning. My students were never the wiser.

Q: From my reading, Unstraight was about music, but also very much about embracing queer identities and speaking up on LGBT and feminist issues. What are your thoughts on how the mathematical profession is doing on these issues?

ER: There have been such enormous strides in public acceptance and awareness of members of the queer community over the past fifteen years that at the moment I'm much more personally concerned about issues confronting students of color and continued biases against femininity in mathematics (which I'm largely insulated from, given my more androgynous aesthetic).

Q: Was electric bass your first instrument? How did you start playing?

ER: I started playing viola in elementary school and played in an orchestra through college. In grad school, I spent more time playing fingerstyle guitar, which is quiet enough to practice in an apartment building even when I got home late at night. In my last few months at Chicago, I formed my first band, Riehl Mann, with Katie Mann, a geometric group theorist, which started because I was looking for an excuse to play her cello. We bought beer to bribe our friends to listen to us play as many instruments as we could manage (e.g., to cover *The Royal Tenenbaums* album) in a couple of house shows.

I realized that I could be in a much better band as a bassist than as a guitarist, so I advertised myself that way when I moved back to Cambridge for my postdoc, even though I didn't own a bass at the time. I practiced for my audition with Unstraight by playing just the bottom four strings on the guitar, and worked out how to play the bass line in The Blow's "True Affection," a particular favorite of our lead singer. That, plus the fact that I could visually recognize the chords the guitarist was playing to pick out the bass note was enough to get me the spot.

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Q: How did you get started playing rugby and football, and how does football fit into your life now?

ER: I started playing rugby my freshman year in college and continued for seven years, through Part III and Cambridge, and the first half of my PhD in Chicago; my PhD advisor, Peter May, was horrified when I broke my arm at the end of my first year. Then I spent 4.5 months in Sydney visiting the famous Centre of Australian Category Theory and fell in love with Australian Rules Football, which I've played for the past seven years. This is actually why I'm in Melbourne at the moment. Since 2010 I've been a member of the USA Freedom, which is the (somewhat awkward) name of the US women's national team in Australian Rules football. There are tryouts every year, mostly just to play a test match against Canada, but every three years they hold a two week "International Cup": http://www.afl.com.au/internationalcup

Q: What is next for you, in math and life?

ER: One of my favorite things about academia is that the job changes all the time, or at least it can, if you want it to. Right now I'm focused on growing the category theory group at Johns Hopkins and a few long-term research projects that I'd love to get through before an MSRI semester on Higher Categories and Categorification that will take place in 2020. In a decade's time, I hope I'm working on projects that I can't even imagine now and have found a way to be a part of larger mathematical and public conversations.

SCUDEM

SIMIODE, A Systematic Initiative for Modeling Investigations & Opportunities with Differential Equations, announces a new new program for high school and undergraduate students: Student Competition Using Differential Equation Modeling—SCUDEM. This competition is for three-member student teams. SCUDEM takes place over the week of April 16–21, 2018. Teams will work initially at their home institution, developing approaches and solutions to one of three posed modeling scenarios. These are designed so that every team may experience success in modeling, enhance their model building skills, and increase their confidence in modeling with differential equations. On Competition Saturday, teams will travel with their faculty coach to a nearby host site to complete the projects and engage in other activities. For complete details visit www.simiode.org/scudem.

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