1. **Ch 1 51.1 basic models, direction fields.**

**Rates of change → derivative**

**Differential Equations**: equations containing derivatives

**Examples**: motion of objects (fluids), flow of current in electric circuits, dissipation of heat, increase/decrease of populations/Money.

**Mathematical models**: a DE that describes some physical process (or a system of DEs)

**Example 1**: (Motion of a falling object) velocity.

- Newton's second law: \( F = ma \), \( a = \frac{dv}{dt} \).
  - net force \( \), mass \( \), acceleration \( \) (m/s²)
  - in Newtons \( \), kg \( \), m/s²

\[
Force = mg - \nu v
\]

\[
\Rightarrow m \frac{dv}{dt} = mg - \nu v
\]

- \( \nu = 10 \text{ kg/s} \), \( v = 2 \text{ kg/s} \) then \( \frac{dv}{dt} = 9.8 - \frac{v}{5} \)

**Direction Field**: first let us consider what information can be obtained directly from the DE itself.

- Given a value of \( \nu(t_0) \), we have \( \frac{dv}{dt}(t_0) \)

**Motivation**: perspective of DE

**Objectives**

- DE & direction fields
  - velocity: \( \nu = g - \frac{\nu}{m} \)
  - population: \( p' = rp - k \)

Equilibrium: \( \frac{dv}{dt} = 0 \) at \( v = 49 \).
Definition: Direction fields. The values of \( f(t,y) \) in the \( t-y \) plane as slope/direction.

\[
\frac{dy}{dt} = f(t,y)
\]

**Example 2.** (Population of field mice in presence of owls)

- Increase at a rate proportional to the current population \( P(t) \).

\[
\frac{dp}{dt} = rP \quad r = \text{rate constant in months}
\]

- Killed by owls \( k=450 \) per month.

\[
\frac{dp}{dt} = rP - k \quad r = 0.5, \quad k = 450
\]

- Equilibrium: \( P(t) = \frac{k}{r} = 900 \).

\[
\begin{align*}
\text{If } P(0) > 900, \quad & \text{then } P(t) \to +\infty \quad \text{as } t \to \infty \\
\text{if } P(0) < 900, \quad & \text{then } P(t) \to -\infty \quad \text{as } t \to \infty
\end{align*}
\]

**Direction fields**

- Example: Draw a direction field of \( y' = y(4-y) \).

<table>
<thead>
<tr>
<th>( y' )</th>
<th>( y = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
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</tr>
<tr>
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<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>-5</td>
<td>5</td>
</tr>
</tbody>
</table>

**Solution:** Equilibrium: \( 0 = y' = y(4-y) \) \( \Rightarrow \) \( y = 0 \) or \( y = 4 \).

- Reading: Constructing mathematical models.

- HW: 8.1 # 6, 14, 15-20