LECTURE NOTES FROM WEEK 6

1. Tuesday, Oct 16, 2007

1. New concepts:
   (1) **local max** (peaks) and **local min** (valleys)
   *Note*. In general, boundary points (edges of interval) are not local extrema (even if they can be global extrema)
   (2) **global max/min** (over an interval): points where a function achieves its max/min
   (3) **critical points**: where \( f'(x) = 0 \)
   *Note*. We can think of the critical points as "determined", since they are the roots of an explicit equation (that one can, in applications, solve).

2. **Theorem (Extreme Values Theorem)**. A continuous function \( f : [a, b] \rightarrow \mathbb{R} \) has global max/min.
   *Note*. If \( c_{\text{max}} \) is the point of global max, then \( f(c_{\text{max}}) = \max_{[a, b]} f \) (the maximum value of \( f \) over the interval \([a, b]\)).

3. **Theorem (Fermat)**. A point of local max (local min) for a differentiable function is also a critical point.

   **Application.** Determine \( \max_{[a, b]} f \) over a closed interval \([a, b]\) (when \( f \) is differentiable).

   **Answer.** You have to make a table
   \[
   \begin{array}{c|c|c}
   x & \text{critical points} & \text{boundary points} \\
   \hline
   f(x) & & \\
   \end{array}
   \]

   Similar approach if you have to determine \( \min_{[a, b]} f \).

2. Wednesday, Oct 17, 2007

1. **Proof of Fermat’s theorem.**

2. **Theorem (Rolle’s Theorem)**

   **Given:**
   - \( f(x) \) differentiable function
   - \( a < b \) such that \( f(a) = f(b) \)

   **Then:**
   - \( \exists c \in (a, b) \) such that \( f'(c) = 0 \)

   *Note*. We discussed a proof of Rolle’s theorem.

   **Application.** Prove that a certain equation has a unique root.

   The answer is a combination of
   - IVT (to prove that at least one root exists)
   - proof by contradiction (which uses Rolle’s Theorem).

3. **Theorem. (MVT=Mean Value Theorem)**

   **Given:**
   - \( f(x) \) differentiable function
   - \( a < b \)

   **Then:**
   - \( \exists c \in (a, b) \) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \)

   **Application.** \( \sin(x) \leq x, \forall x > 0. \)