HOMEWORK 9: DUE WEDNESDAY, APRIL 2

1. TRIPLE INTEGRALS

Definition. For a solid region $W \subset \mathbb{R}^3$, we define the volume of $W$ as

$$\text{vol}(W) = \int \int \int_W \, dx\,dy\,dz$$

(the integral of $f(x, y, z) = 1$ over $W$)


1.2. Compute the volume of the sphere of radius $R$ using Cavalieri’s Principle.

1.3. Compute the volume of the solid region enclosed between the sphere $x^2 + y^2 + z^2 = 9$ and the planes $x = 2$ and $x = 3$.

1.4. Find the volume of the solid region bounded by $x^2 + 2y^2 = 2$, $z = 0$, and $x + y + 2z = 2$ (This is exercise 10, p. 364.).

2. AVERAGES

For a function $f(x, y, z) : \mathbb{R}^3 \to \mathbb{R}$ and $W \subset \mathbb{R}^3$ a solid domain, we define the average of $f$ over $W$ as:

$$\text{avg}(f) = \frac{1}{\text{vol}(W)} \int \int \int_W f(x, y, z) \, dx\,dy\,dz$$

2.1. Let $x_W = \text{avg}(x)$, $y_W = \text{avg}(y)$, $z_W = \text{avg}(z)$ the averages of $x, y, z$ over the solid region $W$ from Example 5, p. 361 of the textbook (this is the example discussed in class). Find $x_W, y_W, z_W$ (you can use part of the classroom computations for this exercise).

Note. The point of coordinates $(x_W, y_W, z_W)$ is the center of mass of the solid region $W$.

THAT’S IT!