1. Vector geometry

1.1. Find the equation of the plane that passes through \(A(1, 2, 0), B(0, 1, -2), C(4, 0, 1)\).

1.2. Find the distance from the point \((2, 1, -2)\) to the plane \(x - 2y + 2z + 5 = 0\).

1.3. Given two vectors \(a\) and \(b\), do the two equations
\[
\mathbf{v} \times \mathbf{a} = \mathbf{b}, \quad \mathbf{v} \cdot \mathbf{a} = \|\mathbf{a}\|
\]
determine the vector \(\mathbf{v}\) uniquely? Argue both geometrically and analytically.

1.4. Let \(A = (1, 2, 3)\) and \(B = (-1, 3, 2)\) two points in space. The point \(M\) on the line segment \(AB\) is such that \(|AM| = 4|MB|\). Determine the coordinates of \(M\).

1.5. A ship at position \((1, 0)\) sights a rock at position \((2, 4)\). What is the vector joining the ship to the rock? What angle \(\theta\) does it make with due north?

1.6. A 1-kg mass located at the origin is suspended by ropes attached to the two points \((1, 1, 1)\) and \((-1, -1, 1)\). If the force of gravity is pointing in the direction of the vector \(-k\), what is the vector describing the force along each rope? [A 1-kg mass weighs 9.8 N (newtons).]

1.7. Find the line through \((3, 1, -2)\) that intersects and is perpendicular to the line \(x = -1 + t, y = -2 + t, z = -1 + t\).

1.8. Find the equation of the line that passes through the point \((1, -2, -3)\), and is perpendicular on the plane \(3x - y - 2z + 4 = 0\).

1.9. a) Calculate \((i - j + k) \cdot (-i - k + \frac{1}{2}k)\).
b) Find the length of \(\mathbf{v} = 2i - 6j + 7k\).
c) Find the values of \(c\) such that \(\|i + j + ck\| = 4\).
d) Show that by dividing a (nonzero) vector by its length one obtains a unit vector.

1.10. Find the distance from \((2, 8, -1)\) to the line that passes through \((1, 1, 1)\) in the direction of \((1/\sqrt{3})i + (1/\sqrt{3})j + (1/\sqrt{3})k\).

1.11. Find the parametric equation of the line where the planes \(x + y - z = 0\) and \(x - y - z = 0\) meet.

1.12. a) Find the equation of the plane that passes through \(A = (1, 2, 0), B = (0, 1, -2)\) and \(C = (4, 0, 1)\).
b) Find the area of the triangle \(ABC\).

1.13. The curve \(\mathbf{c}(t) = (t, t^2, t^3)\) intersects the plane \(4x + 2y + z = 24\) in a single point. Find the coordinates of that point and calculate the (cosine of the) angle between the velocity of \(\mathbf{c}\) at that point and the normal vector to the plane.

1.14. Where does the plane tangent to \(z = e^{x-y}\) at \((1, 1, 1)\) meet the \(z\)-axis?
1.15. A particle following the path \( c(t) = (t^2, t^3 - 4t, 0) \) flies off in the tangent direction at \( t_0 = 2 \). Determine its position at time \( t_1 = 3 \).

1.16. A particle following the path \( c(t) = (\cos(\pi t), \sin(\pi t), \pi t) \) flies off in the tangent direction at time \( t_0 = 9/4 \). Determine how long does it take for the particle to hit the wall given by equation \( x = 0 \).

1.17. Functions.

1.18. Let \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) given by:

\[
  f(x, y) = \begin{cases} 
  \frac{x^2 y}{x^2 + y^4}, & (x, y) \neq (0, 0), \\
  0, & (x, y) = (0, 0)
  \end{cases}
\]

Consider two paths in \( \mathbb{R}^2 \) given by: \( c_1(t) = (-t, t) \) and \( c_2(t) = (t^2, t) \).

a) Sketch the paths \( c_1 \) and \( c_2 \).

b) Compute \( \lim_{t \to 0} f(c_1(t)) \) and \( \lim_{t \to 0} f(c_2(t)) \).

c) Is \( f \) continuous at \((0, 0)\)? Justify your answer.

1.19. For \( V(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \), compute \( \nabla V \).

1.20. Calculate \( \nabla h(1, 1, 1) \) for \( h(x, y, z) = (x + z)e^{x-y} \).

1.21. Give an estimate to the following quantity

\[
(0.99e^{0.02})^8
\]

using a linear approximation.

1.22. Compute the matrix of partial derivatives of \( f(x, y) = (e^x, \sin(xy)) \).

1.23. True or false: if \( \frac{\partial d}{\partial x}(1, 0) \) and \( \frac{\partial f}{\partial y}(1, 0) \) exist, then \( f \) is differentiable at \((1, 0)\).

1.24. Is the function \( f(x, y) = x^{1/3}y^{1/3} \):

a) continuous

b) differentiable

Justify your answers.

2. Chain rule

2.1. State the chain rule in full generality.

2.2. Suppose the temperature at the point \((x, y, z)\) in space is \( T(x, y, z) = x^2 + y^2 + z^2 \). Let a particle follow the right circular helix \( \sigma(t) = (\cos t, \sin t, t) \), and let \( T(t) \) its temperature at time \( t \). Compute \( T'(t) \) using the chain rule.

2.3. In general the trajectory of a particle in space is given by a map \( c(t) = (x(t), y(t), z(t)) : \mathbb{R} \rightarrow \mathbb{R}^3 \).

a) Use the chain rule to compute \( \frac{d}{dt} \|c(t)\|^2 \).

b) What can you say about the \( \|c(t)\| \) if the particle moves across a sphere centered at the origin?

c) In the situation described at b), show that the velocity of the particle is always normal to the position vector: \( c'(t) \perp c(t) \), for all \( t \).
2.4. Let \( f : \mathbb{R} \to \mathbb{R} \) differentiable and \( h(x, y) = f\left(\frac{x + y}{x - y}\right) \). Prove that \( h(x, y) \) satisfies the equation \( x \frac{\partial h}{\partial x} + y \frac{\partial h}{\partial y} = 0 \).