

## PRACTICE PROBLEMS: SERIES

1. Assume  $a_n$  are such that  $\sum_{n=1}^{\infty} 2^n a_n$  is convergent.
  - a) True or false:  $\sum_{n=1}^{\infty} a_n$  is convergent.
  - b) True or false:  $\sum_{n=1}^{\infty} (-1)^n 2^n a_n$  is convergent.
  - c) True or false:  $\sum_{n=1}^{\infty} n a_n$  is convergent.
2. Determine the radius and domain of convergence for  $\sum_{n=0}^{\infty} 2^n x^{n^2}$ .
3. a) Determine the radius and domain of convergence for  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ .  
b) Let  $f(x)$  the sum of the series from a). Prove that  $f$  satisfies the equation  $f''(x) = -f(x)$  on the domain of convergence.
- 4\*. Find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n+1}$ .
- 5\*. Prove Abel's theorem.