0.1. Consider the following elements in $G = SU(3)$: $w_{jk} = E_{jk} - E_{kj} + \sum_{l \neq j, k} E_{ll}$, for $j \neq k$. For example

$$w_{31} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Prove that for any representation $(\pi, V)$ of $SU(3)$

$$\pi(w_{jk}) : V(\lambda) \rightarrow V(s_{jk}\lambda)$$

where $s_{jk} : it^* \rightarrow it^*$ is the reflection across the line $\alpha_{jk}$. In particular, $V(\lambda)$ and $V(s_{jk}\lambda)$ have the same dimension.

0.2. Assume $\pi$ is an irreducible representation of $SU(3)$ and $\lambda_0 = a_1l_1 + a_2l_2 + a_3l_3$ is its highest weight. We have seen that the weights $E(\pi)$ occur inside a (hexagonal) polygon with one of the vertices at $\lambda_0$. Prove that the other vertices occur at the weights

$$\lambda' = a_{\sigma(1)}l_1 + a_{\sigma(2)}l_2 + a_{\sigma(3)}l_3$$

where $\sigma$ varies over the permutations of the set $\{1, 2, 3\}$.

0.3. Let $\pi_{st}$ the standard representation of $SU(3)$ on $V = \mathbb{C}^3$ (by matrix multiplication on the left). Determine the weights of $\pi_{st}$ and express them in terms of $l_1, l_2, l_3$.

0.4. Same question as above for the representation $\text{sym}^2 \pi_{st}$ of $SU(3)$ on $\text{sym}^2 V$. Argue that $\text{sym}^2 \pi_{st}$ is irreducible and determine its highest weight.