

## MATH 423 - HOMEWORK 5

### 1. PART I

- 1.1. Assume  $c(t) : \mathbb{R} \rightarrow GL(n, \mathbb{C})$  is a continuous one-parameter subgroup. Prove that there exist  $X \in gl(n, \mathbb{C})$  such that  $c(t) = \exp(tX)$ ,  $\forall t \in \mathbb{R}$ .
- 1.2. Determine all continuous groups homomorphisms  $c : \mathbb{R} \rightarrow \mathbb{C}^\times$ . Identify the unitary ones.
- 1.3. For  $U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $H = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \in sl(2, \mathbb{R})$ , compute  $\exp(tU)$  and  $\exp(tH)$ .

### 2. PART II

- 2.1. Prove that

$$\lim_{n \rightarrow \infty} (\exp(X/n) \exp(Y/n) \exp(-X/n) \exp(-Y/n))^{n^2} = \exp([X, Y])$$

for  $X, Y \in gl(n, \mathbb{C})$ .