MATH 423: HOMEWORK 7

1. Part I

1.1. Assume G is a connected Lie group. Prove that $Z(G) := \{g \in G : gx = xg, \forall x \in G\}$ is a Lie subgroup with Lie algebra $z(\mathfrak{g}) := \{Z \in \mathfrak{g}, [Z, X] = 0, \forall X \in \mathfrak{g}\}$.

1.2. Assume G is a connected Lie group and $\Delta \lhd G$ is a normal, discrete subgroup. Prove that $\Delta \subset Z(G)$.

1.3. Assume K is a compact abelian Lie group such that K/K^0 is cyclic. Prove that K has a topological generator.

2. Part II

2.1. Assume (π, V) is a finite dimensional representation of S^1. Prove that $V = \oplus_n V(n)$, where $V(n) = \{v \in V : \pi(e^{i\theta})v = e^{in\theta}v, \forall\, \theta \in \mathbb{R}\}$.

2.2. Prove that a compact abelian Lie group has a (continuous) group character of finite order $\chi : G \to \mathbb{C}^\times$ if and only if G is disconnected.