ABSTRACT

Let $X = SL(2,\mathbb{Z})\backslash\mathbb{H}$ be the modular surface. We consider the Eisenstein series with unitary parameter $E(z, \frac{1}{2} + it)$. We show that, when restricted to a fixed compact subset $\Omega \subset X$, the L^4 norm $\|E(\frac{1}{2}+it)\|_{L^4(\Omega)}$ is $O\big(\sqrt{\log t}\big)$. On the other hand, it is known from the work of Luo and Sarnak that $\|E(\frac{1}{2}+it)\|_{L^2(\Omega)}$ is asymptotically equal to $c_\Omega\sqrt{\log t}$. This shows that, in the continuous spectrum, the (generalized) eigenfunctions of the hyperbolic Laplace operator have bounded L^4 norm in the high energy limit, after an appropriate normalization. This is in accord with the conjectured behavior of eigenfunctions in the quantization of a classically chaotic system.

In the case of an arithmetic surface we reduce the L^4 norm problem, via triple product identities, to questions about a *family sum* of automorphic L-functions; techniques from analytic number theory can then be applied successfully to establish a sharp estimate for the L^4 norm.