1. Compute the following limits if they exists:

(a) [6 pts] For $f : \mathbb{R}^n \to \mathbb{R}$ and $x \in \mathbb{R}^n$, we define $f(x) = \|x\|$. What is $\lim_{x \to x_0} f(x)$?

(b) [9 pts] $\lim_{(x,y) \to (0,0)} \sqrt{\frac{2x+y}{x-2y}}$ with $x \neq 2y$.

2. [10 pts] For the function

$$f(x, y, z) = (xy)^{\frac{1}{z}}$$

and

$$g(t) = (2t, t^2, t^3),$$

find $\nabla f$ and $g'$ and evaluate $(f \circ g)'(1)$.

3. [10 pts] Find the equation for the plane tangent to the surface

$$z = \ln \sqrt{2 + xy}$$

at the point $(1, -1, 0)$.

4. [10 pts] Determine the second-order Taylor formula for the function

$$f(x, y) = \sin(xy) + \cos(xy)$$

about the point $(0, 0)$.

5. [10 pts] Find the critical points of the function

$$f(x, y) = y \sin(\pi x)$$

and then determine whether they are local maxima, local minima, or saddle points.

6. [15 pts] Find the absolute maximum and minimum for the function

$$f(x, y) = xy - y + x - 1$$

on the set

$$B = \{(x, y) \mid x^2 + y^2 \leq 2\}.$$

7. [10 pts] A rectangular box with no top is to have a surface area $16 \text{ m}^2$. Find the dimensions that maximize its volume.
8. **[10 pts]** Is it possible to solve the system of equations

\[
\begin{align*}
xy^2 + xzv + yv^2 &= 3 \\
u^3yz + 2xv - u^2v^2 &= 2
\end{align*}
\]

for \(u(x, y, z), v(x, y, z)\) near \((x, y, z) = (1, 1, 1)\) and \((u, v) = (1, 1)\)? If yes, compute \(\frac{\partial v}{\partial y}\) at \((x, y, z) = (1, 1, 1)\).

9. **[10 pts]** When \(c \neq 0\), the level curve \(\{ (x, y) \mid f(x, y) = c \}\) of the function

\[f(x, y) = \frac{2x}{x^2 + y^2}\]

is a circle. What is its radius, and where is its center? What happens when \(c = 0\)?