1.1 #4 Determine whether the statement is true or false.

[Solution]
The answer is true because
\[
\frac{-5}{6} = -\frac{10}{12} < -\frac{11}{12}.
\]

1.1 #6 Show the interval \((-2.5]\) on the number line.

[Solution]
The interval on the number line looks like
\((-2.5]\)

1.1 #10 Show the interval \((-\infty, 0.5]\) on the number line.

[Solution]
The interval on the number line looks like
\((-\infty, 0.5]\)

1.1 #20 Find the value of \(x\) that satisfy the inequalities
\[x - 4 \leq 1 \text{ and } x + 3 > 2.\]

[Solution]
\(x - 4 \leq 1\) implies \(x \leq 5\). \(x + 3 > 2\) implies \(x > -1\). Thus, we have
\[-1 < x \leq 5.\]

1.1 #30 Evaluate the expression \(2\sqrt{3} - 3 - |\sqrt{3} - 4|\).

[Solution]
Since \(2\sqrt{3} \approx 3.4641 > 3\), we have \(|2\sqrt{3} - 3| = 2\sqrt{3} - 3\). Since \(\sqrt{3} = 1.7321 < 4\), we have \(|\sqrt{3} - 4| = -(\sqrt{3} - 4) = 4 - \sqrt{3}\). Thus,
\[
\begin{align*}
|2\sqrt{3} - 3| - |\sqrt{3} - 4| &= \left(2\sqrt{3} - 3\right) - \left(-\left(\sqrt{3} - 4\right)\right) \\
&= 2\sqrt{3} - 3 + \sqrt{3} - 4 \\
&= 3\sqrt{3} - 7.
\end{align*}
\]
1.1 #34 Suppose \( a \) and \( b \) are real numbers other than zero and that \( a > b \). State whether the inequality
\[
\frac{1}{a} > \frac{1}{b}
\]
is true or false.

[Solution]
The answer is false because \( \frac{1}{a} > \frac{1}{b} \) implies \( b > a \).

1.1 #38 Determine whether the statement
\[
|b^2| = b^2
\]
is true for all real numbers \( a \) and \( b \).

[Solution]
The answer is true because \( b^2 \geq 0 \) for all real number \( b \) so that \( |b^2| = b^2 \) by the definition of absolute value.

1.1 #56 Evaluate the expression
\[
\left( \frac{9^{-3} \cdot 9^5}{9^{-2}} \right)^{-\frac{1}{2}}.
\]

[Solution]
We have
\[
\left( \frac{9^{-3} \cdot 9^5}{9^{-2}} \right)^{-\frac{1}{2}} = \left( \frac{9^{-3+5}}{9^{-2}} \right)^{-\frac{1}{2}} = \left( \frac{9^2}{9^{-2}} \right)^{-\frac{1}{2}} = (9^{2-(-2)})^{-\frac{1}{2}}
\]
\[
= (9^4)^{-\frac{1}{2}} = \frac{1}{(9^4)^{\frac{1}{2}}} = \frac{1}{9^2} = \frac{1}{81}.
\]

1.1 #90 Simplify the expression
\[
\sqrt[3]{27r^6} \cdot \sqrt{s^2t^4}.
\]
(Assume that \( r, s \) and \( t \) are positive.)

[Solution]
We have
\[
\sqrt[3]{27r^6} \cdot \sqrt{s^2t^4} = (27r^6)^{\frac{1}{3}} \cdot (s^2t^4)^{\frac{1}{2}} = (27)^{\frac{1}{3}} \cdot (r^6)^{\frac{1}{3}} \cdot (s^2)^{\frac{1}{2}} \cdot (t^4)^{\frac{1}{2}}
\]
\[
= 3 \cdot r^2 \cdot s^1 \cdot t^2 = 3r^2st^2.
\]

1.1 #114 Celsius and Fahrenheit Temperatures The relationship between Celsius (\( ^\circ C \)) and Fahrenheit (\(^\circ F\)) temperatures is given by the formula
\[
C = \frac{5}{9}(F - 32).
\]
(a) If the temperature range for Montreal during the month of January is \(-15^\circ < C < -5^\circ\), find the range in degrees Fahrenheit in Montreal for the same period.
(b) If the temperature range for New York City during the month of June is \(63^\circ < F < 80^\circ\), find the range in degrees Celsius in New York City for the same period.

[Solution]
(a) Since \(-15 < C < -5\), we have \(-15 < \frac{5}{9}(F - 32) < -5\). So,

\[
-15 < \frac{5}{9}(F - 32) < -5 \\
\implies \quad -15 \cdot \frac{9}{5} < F - 32 < -5 \cdot \frac{9}{5} \\
\implies \quad -27 < F - 32 < -9 \\
\implies \quad -27 + 32 < F < -9 + 32 \\
\implies \quad 5 < F < 23.
\]

(b) Since \(63 < F < 80\), we have

\[
63 < F < 80 \\
\implies \quad 63 - 32 < F - 32 < 80 - 32 \\
\implies \quad 31 < F - 32 < 48 \\
\implies \quad \frac{5}{9} \cdot 31 < \frac{5}{9}(F - 32) < \frac{5}{9} \cdot 48 \\
\implies \quad \frac{155}{9} < C < \frac{240}{9}.
\]

\[\boxed{}\]

1.1 #118 Quality Control The diameter \(x\) (in inches) of a batch of ball bearings manufactured by PAR manufacturing satisfies the inequality

\[|x - 0.1| \leq 0.01.\]

What is the smallest diameter a ball bearing in the batch can have? The largest diameter?

[Solution]

If \(x - 0.1 \geq 0\), we have \(x - 0.1 \leq 0.01\). This implies that \(x \leq 0.01 + 0.1 = 0.11\).

If \(x - 0.1 < 0\), we have \(-(x - 0.1) \leq 0.01\). This implies that \(x - 0.1 \geq -0.01\) which is \(x \geq 0.09\).

So, \(0.09 \leq x \leq 0.11\). Thus, the smallest diameter is 0.09 inches and the largest diameter is 0.11 inches.

[Alternative Solution]

Note that \(|x - 0.1| \leq 0.01\) implies \(-0.01 \leq x - 0.1 \leq 0.01\). So, we have we have

\[-0.01 \leq x - 0.1 \leq 0.01\]

\[
\implies \quad -0.01 + 0.1 \leq x \leq 0.01 + 0.1 \\
\implies \quad 0.09 \leq x \leq 0.11.
\]

\[\boxed{}\]
1.2 #20 Perform the indicated operations and simplify the expression

\[(x^\frac{1}{2} + 1) \left( \frac{1}{2} x^{-\frac{1}{2}} \right) - (x^\frac{1}{2} - 1) \left( \frac{1}{2} x^{-\frac{1}{2}} \right).\]

**[Solution]**

\[
\begin{align*}
(x^\frac{1}{2} + 1) \left( \frac{1}{2} x^{-\frac{1}{2}} \right) & - (x^\frac{1}{2} - 1) \left( \frac{1}{2} x^{-\frac{1}{2}} \right) \\
= & \left( \frac{1}{2} x^{-\frac{1}{2}} \right) \left[ (x^\frac{1}{2} + 1) - (x^\frac{1}{2} - 1) \right] \\
= & \left( \frac{1}{2} x^{-\frac{1}{2}} \right) 2 \\
= & x^{-\frac{1}{2}}.
\end{align*}
\]

1.2 #32 Factor the expression

\[3x^3 - x^2 + 3x - 1.\]

**[Solution]**

\[
\begin{align*}
3x^3 - x^2 + 3x - 1 \\
= & \ x^2 (3x - 1) + 3x - 1 \\
= & \ (3x - 1) (x^2 + 1).
\end{align*}
\]

1.2 #42 Factor the expression

\[8a^2 - 2ab - 6b^2.\]

**[Solution]**

\[
\begin{align*}
8a^2 - 2ab - 6b^2 \\
= & \ 8a^2 - 8ab + 6ab - 6b^2 \\
= & \ 8a (a - b) + 6b (a - b) \\
= & \ (8a + 6b) (a - b) \\
= & \ 2 (4a + 3b) (a - b).
\end{align*}
\]

**[Alternative Solution]**

Treat \(a\) as a variable and \(b\) as a constant. Then, we can pretend that \(8a^2 - 2ab - 6b^2\) is a quadratic formula. To factor it, use the cross table

\[
\begin{array}{c|c}
4 & 3b \\
\hline
2 & -2b
\end{array}
\]

to get that \(8a^2 - 2ab - 6b^2 = (4a + 3b) (2a - 2b) = 2 (4a + 3b) (a - b).\)

1.2 #58 Find the real roots of the equation

\[\frac{1}{2}a^2 + a - 12 = 0\]

by factoring.

**[Solution]**
Note that the roots of $\frac{1}{2}a^2 + a - 12 = 0$ is the same as $a^2 + 2a - 24 = 0$. (Why?) The leading coefficient is $1$ so we can expect that we can factor our quadratic formula $a^2 + 2a - 24$ into $(a + m)(a + n)$. Thus, we have

$$a^2 + 2a - 24 = (a + m)(a + n) = a^2 + (m + n)a + mn.$$  

By comparing the corresponding coefficients, we have $m + n = 2$ and $mn = -24$. Since $mn = -24$, we have the pair $(m,n)$ could be one of the following: $(1, -24)$, $(2, -12)$, $(3, -8)$, $(4, -6)$, $(1, 24)$, $(-2, 12)$, $(-3, -8)$, $(-4, 6)$. Because $m + n = 2$, only $(-4, 6)$ satisfies our conditions. Therefore, $a^2 + 2a - 24 = (a - 4)(a + 6)$. By factoring, the roots are $4$ and $-6$.

1.2 #74 Perform the indicated operations and simplify the expression

$$\frac{3x^2 - 4xy - 4y^2}{x^2y} \div \frac{(2y - x)^2}{x^3y}.$$  

[Solution]

$$\frac{3x^2 - 4xy - 4y^2}{x^2y} \div \frac{(2y - x)^2}{x^3y} = \frac{3x^2 - 4xy - 4y^2}{x^2y} \times \frac{x^3y}{(2y - x)^2} = \frac{3x^2 - 4xy - 4y^2}{(2y - x)^2} \times \frac{x^3y}{x^2y} = \frac{(x - 2y)(3x + 2y)}{(2y - x)^2} \times \frac{3x^2 - 4xy - 4y^2}{x^3y} = \frac{-(3x + 2y)}{(2y - x)} \times \frac{x}{x - 2y} = \frac{-x(3x + 2y)}{(2y - x)} = \frac{x(3x + 2y)}{x - 2y}.$$  

Note that we can use the similarly idea in 1.2 #58 to factor $3x^2 - 4xy - 4y^2$. Or, use the cross table

$$\begin{array}{c|c}
   x & -2y \\
   \hline
   3x & 2y \\
\end{array}$$

to get $3x^2 - 4xy - 4y^2 = (x - 2y)(3x + 2y)$.

1.2 #82 Perform the indicated operations and simplify the expression

$$\frac{1}{x} + \frac{1}{y}.$$  

[Solution]

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y + x}{xy} = \frac{(xy)(y + x)}{xy(xy - 1)} = \frac{y + x}{xy - 1}.$$  

1.2 #94 Rationalize the denominator of the expression

$$\frac{2\sqrt{a} + \sqrt{b}}{2\sqrt{a} - \sqrt{b}}.$$

1.2 #82 Perform the indicated operations and simplify the expression

$$\frac{1}{x} + \frac{1}{y}.$$  

[Solution]

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y + x}{xy} = \frac{(xy)(y + x)}{xy(xy - 1)} = \frac{y + x}{xy - 1}.$$  

1.2 #94 Rationalize the denominator of the expression

$$\frac{2\sqrt{a} + \sqrt{b}}{2\sqrt{a} - \sqrt{b}}.$$
[Solution]
To rationalize the denominator, we need to multiply $2\sqrt{a} + \sqrt{b}$ for both numerator and denominator. We have

\[
\frac{2\sqrt{a} + \sqrt{b}}{2\sqrt{a} - \sqrt{b}} = \frac{(2\sqrt{a} + \sqrt{b}) (2\sqrt{a} + \sqrt{b})}{(2\sqrt{a} - \sqrt{b}) (2\sqrt{a} + \sqrt{b})} = \frac{(2\sqrt{a})^2 + 2\sqrt{a}\sqrt{b} + \sqrt{b})^2}{(2\sqrt{a})^2 - (\sqrt{b})^2}
\]

\[
= \frac{4a + 4\sqrt{ab} + b}{4a - b}.
\]