MA119-A Applied Calculus for Business

2006 Fall

Homework 2 Solutions

Due 9/13/2006 10:30AM

1.3 #24 Find the distance between the given points (−2, 1) and (10, 6).

[Solution]
The distance between (−2, 1) and (10, 6) is
\[ d = \sqrt{(10 - (-2))^2 + (6 - 1)^2} = 13. \]

1.3 #28 Show that the triangle with vertices (−5, 2), (−2, 5) and (5, −2) is a right triangle.

[Solution]
The distance between (−5, 2) and (−2, 5) is
\[ d_1 = \sqrt{(-2 - (-5))^2 + (5 - 2)^2} = 3\sqrt{2}. \]
The distance between (−2, 5) and (5, −2) is
\[ d_2 = \sqrt{(5 - (-2))^2 + (-2 - 5)^2} = 7\sqrt{2}. \]
The distance between (5, −2) and (−5, 2) is
\[ d_3 = \sqrt{(-5 - 5)^2 + (2 - (-2))^2} = 2\sqrt{29}. \]
We notice that
\[ d_1^2 + d_2^2 = (3\sqrt{2})^2 + (7\sqrt{2})^2 = 116 = (2\sqrt{29})^2 = d_3^2. \]
By Pythagorean theorem, these three points form a right triangle.

1.3 #34 Find an equation of the circle that satisfies the given conditions: center (−a, a) and radius 2a.

[Solution]
The equation of the circle with center (−a, a) and radius 2a is
\[ (x - (-a))^2 + (y - a)^2 = (2a)^2, \]
that is,
\[ x^2 + y^2 + 2ax - 2ay - 2a^2 = 0. \]

1.3 #36 Delivery Charges A furniture store offers free setup and delivery services to all points within a 25-mi radius of its warehouse distribution center. If you live 20 mi east and 14 mi south of the warehouse, will you incur a delivery charge? Justify your answer.

[Solution]
Consider the warehouse distribution center to be the origin of a Cartesian coordinate system \((xy\text{-plane})\). Treat the east as the positive \(x\) direction and the north as the positive \(y\) direction. So, your house can be labeled as \((20, -14)\). Therefore, the distance between your house and the warehouse distribution center is
\[
\sqrt{(20 - 0)^2 + (-14 - 0)^2} \approx 24.413 < 25.
\]
Thus, you will not incur a delivery charge.

1.3 #38 Minimizing Shipping Costs Refer to the figure for Exercise 37. Suppose a fleet of 100 automobiles are to be shipped from an assembly plant in town \(A\) to town \(D\). They may be shipped either by freight train along Route 1 at a cost of \(22\)¢/mile per automobile or by truck along Route 2 at a cost of \(21\)¢/mile per automobile. Which means of transportation minimizes the shipping cost? What is the net savings?

[Solution]
According to the figure for Exercise 37, the total length of Route 1 is
\[
d_1 = \sqrt{(400 - 0)^2 + (300 - 0)^2} + \sqrt{(1300 - 400)^2 + (1500 - 300)^2} = 2000
\]
and the total length of Route 2 is
\[
d_2 = \sqrt{(800 - 0)^2 + (1500 - 0)^2} + \sqrt{(1300 - 800)^2 + (1500 - 1500)^2} = 2200.
\]
Thus, the cost of shipping along Route 1 is \(2000 \times 0.22 = 440\) (dollars) and the cost of shipping along Route 2 is \(2200 \times 0.21 = 462\) (dollars). Therefore, shipping along Route 1 minimizes the cost with the net saving is \(462 - 440 = 22\) (dollars).

1.3 #42 Ship \(A\) leaves port sailing north at a speed of 25 mph. A half hour later, ship \(B\) leaves the same port sailing east at a speed of 20 mph. Let \(t\) (in hours) denote the time ship \(B\) has been at sea.

(a) Find an expression in terms of \(t\) giving the distance between the two ships.

(b) Use the expression obtained in part (a) to find the distance between the two ships 2 hr after ship \(A\) has left port.

[Solution]
(a) Since \(t\) denotes the time ship \(B\) has been at sea and ship \(A\) leaves a half hour earlier then ship \(B\), we have the time ship \(A\) has been at sea is \(t + \frac{1}{2}\) (in hours).

Put ship \(A\) and ship \(B\) into a \(x\)-\(y\) coordinate system which the origin represents the port.

Since ship \(A\) is sailing at a speed of 25 mph, the distance that ship \(A\) travels is \(25 \times (t + \frac{1}{2})\) miles. Since ship \(A\) is sailing north, ship \(A\) will be in the coordinates \((0, 25 \times (t + \frac{1}{2}))\).

Since ship \(B\) is sailing at a speed of 20 mph, the distance that ship \(B\) travels is \(20t\) miles. Since ship \(B\) is sailing east, ship \(B\) will be in the coordinates \((20t, 0)\).
Thus, the distance between the two ships is
\[
d(t) = \sqrt{(20t - 0)^2 + (0 - 25 \times \left( t + \frac{1}{2} \right))^2}
\]
\[
= \sqrt{400t^2 + 625 \left( t^2 + t + \frac{1}{4} \right)}
\]
\[
= \sqrt{1025t^2 + 625t + \frac{625}{4}}.
\]

(b) Since ship B left the port a half hour later than ship A, we have that ship B has left the port \(2 - \frac{1}{2} = \frac{3}{2}\) hrs. Thus, the distance between the two ships is
\[
d\left( \frac{3}{2} \right) = \sqrt{1025 \left( \frac{3}{2} \right)^2 + 625 \left( \frac{3}{2} \right) + \frac{625}{4}} = 10\sqrt{34}
\]
miles.

1.4 #18 Given the equation \(2x + 3y = 4\), answer the following questions:
(a) Is the slope of the line described by this equation positive or negative?
(b) As \(x\) increases in value, does \(y\) increase or decrease?
(c) If \(x\) decreases by 2 units, what is the corresponding change in \(y\)?

[Solution]
(a) Re-write the equation as \(y = -\frac{2}{3}x + \frac{4}{3}\). The slope is \(-\frac{2}{3}\) which is negative.
(b) By the equation in (a), as \(x\) increases in value, \(y\) decrease.
(c) When \(x = x_0\), we have \(y_0 = -\frac{2}{3}x_0 + \frac{4}{3}\). If \(x\) decreases by 2 units, say \(x_1 = x_0 - 2\).

Then, the corresponding
\[
y_1 = -\frac{2}{3}x_1 + \frac{4}{3} = -\frac{2}{3}(x_0 - 2) + \frac{4}{3} = -\frac{2}{3}x_0 + \frac{4}{3} + \frac{4}{3} = y_0 + \frac{4}{3}.
\]

Thus, the corresponding change in \(y\) is \(\frac{4}{3}\).

1.4 #46 Find an equation of the line that passes through the point \((2, 4)\) and is perpendicular to the line \(3x + 4y - 22 = 0\).

[Solution] The slope of the line \(3x + 4y - 22 = 0\) is \(-\frac{3}{4}\). Since our line is perpendicular to the given line, the slope of our line \(m\) satisfies \(m \times -\frac{3}{4} = -1\). This implies that \(m = \frac{4}{3}\). So, by the point-slope form, the equation is
\[
(y - 4) = \frac{4}{3}(x - 2),
\]
which is,
\[
4x - 3y + 4 = 0.
\]

1.4 #62 Use the results of Exercise 61 to find an equation of a line with the given \(x\)- and \(y\)-intercepts.

[Solution] \[
\frac{x}{3} + \frac{y}{4} = 1.
\]
1.4 #70 Social Security Contributions For wages less than the maximum taxable wage base, Social Security contributions by employees are 7.65% of the employee’s wages.
(a) Find an equation that expresses the relationship between the wages earned \( x \) and the Social Security taxes paid \( y \) by an employee who earns less than the maximum taxable wage base.
(b) For each additional dollar that an employee earns, by how much is his or her Social Security contribution increased? (Assume that the employee’s wages are less than the maximum taxable wage base.)
(c) What Social Security contributions will an employee who earns $35,000 (which is less than the maximum taxable wage base) be required to make?

[Solution]
(a) \( y = 0.0765x \).
(b) By setting \( x = 1 \), we have \( y = 0.0765 \). Thus, the Social Security contribution increases by 7.65 cents for each additional dollar that an employee earns.
(c) \( y = 0.0765 \times 35000 = 2677.5 \). Thus, an employee who earns $35,000 is required to make $2677.5 Social Security contributions.

1.4 #74 Sales Growth Metro Department Store’s annual sales (in millions of dollars) during the past 5 yr were

\[
\begin{array}{cccccc}
\text{Year}, x & 1 & 2 & 3 & 4 & 5 \\
\text{Annual Sales}, y & 5.8 & 6.2 & 7.2 & 8.4 & 9.0 \\
\end{array}
\]

(a) Plot the annual sales \( y \) versus the year \( x \).
(b) Draw a straight line \( L \) through the points corresponding to the first and fifth years.
(c) Drive an equation of the line \( L \).
(d) Using the equation found in part (c), estimate Metro’s annual sales 4 yr from now \( (x = 9) \).

[Solution]
(a) (Skip)
(b) (Skip)
(c) Using the equation of two points, we get
\[
y - 5.8 = \frac{9.0 - 5.8}{5 - 1} (x - 1),
\]
that is,
\[
y = 1.2x + 4.6.
\]
(d) \( y = 1.2 \times 9 + 4.6 = 15.4 \). So, the estimation of the Metro’s annual sales 4 yr from now is 15.4 million dollars.

1.4 #78 Determine whether the statement is true or false: The line with equation \( Ax + By + C = 0 \), \((B \neq 0)\), and the line with equation \( ax + by + c = 0 \), \((b \neq 0)\), are parallel if \( Ab - aB = 0 \).

[Solution] The slope of the first line is \(-\frac{A}{B}\) and the slope of the second line is \(-\frac{a}{b}\). If \(-\frac{A}{B} = -\frac{a}{b}\), then they are parallel. Since the condition \(-\frac{A}{B} = -\frac{a}{b}\) is equivalent to \(Ab - aB = 0\), we can conclude that these two lines are parallel if \(Ab - aB = 0\).
1.4 #80 Determine whether the statement is true or false: The lines with equations \( ax+by+c_1 = 0 \) and \( bx - ay + c_2 = 0 \), where \( a \neq 0 \) and \( b \neq 0 \), are perpendicular to each other.

[Solution]

The slope of the first line is \( -\frac{a}{b} \) and the slope of the second line is \( \frac{b}{a} \). The multiplication of these two slope is \(-1\). So, they are perpendicular.

1.4 #84 Prove that if a line \( L_1 \) with slope \( m_1 \) is perpendicular to a line \( L_2 \) with slope \( m_2 \), then \( m_1 m_2 = -1 \).

[Solution]

By the figure in the textbook page 44, we have

\[
m_1 = \frac{b-0}{1-0} = b
\]

and

\[
m_2 = \frac{c-0}{1-0} = c.
\]

The length of the line segment \( AO \) is \( \sqrt{(1-0)^2 + (b-0)^2} = \sqrt{b^2+1} \). The length of the line segment \( BO \) is \( \sqrt{(1-0)^2 + (c-0)^2} = \sqrt{c^2+1} \). The length of the line segment \( AB \) is \( \sqrt{(c-b)^2 + (1-1)^2} = |c-b| \). By Pythagorean theorem, we have \( (\sqrt{b^2+1})^2 + (\sqrt{c^2+1})^2 = (c-b)^2 \). This implies that \( bc = -1 \), that is, \( m_1 m_2 = -1 \).