

3. (optional) In this exercise, we will show that $A_5$ is a simple group. Recall that $A_5$ is generated by 3-cycles. Suppose $N$ is a normal group of $A_5$

1) Suppose that $\sigma = (abcde) \in N$. Let $\alpha = (ab)(cd) \in A_5$. Show that $\sigma' = \alpha \sigma \alpha^{-1} = (adceb)$, and $\sigma \sigma' = (acc)$

2) Suppose that $\sigma = (ab)(cd) \in N$. Let $\beta = (abe)$. Show that $\sigma' = \beta \sigma \beta^{-1} = (be)(cd)$, and $\sigma \sigma' = (abe)$.

3) Suppose that $\sigma = (abc) \in N$, and $\sigma_1 = (acd), \sigma_2 = (ade)$. Let $\gamma_1 = (ac)(bd), \gamma_2 = (bd)(ce)$. Show that $\sigma_1 = \gamma_1 \sigma \gamma_1^{-1}$.

4) Show that $A_5$ is simple.

4. (optional*) Let $G$ be a group, $G'$ the set of commutators of $G$, i.e.,

$G' = \{x^{-1}y^{-1}xy | \forall x, y \in G\}$.

Could you give an example of $G$, such that $G'$ is not a subgroup of $G$. (If you want to find $G$ with $|G| < +\infty$, the smallest group having this feature is of order 96.)