This exam contains 10 pages (including this cover page) and 4 problems. Check to see if any pages are missing.

Statement of Ethics

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature __________________________ Date __________________________

1 Fuentes T 1:30 - 2:20 Bloomberg 278
2 Quinan T 3:00 - 3:50 Hodson 301
3 Quinan T 4:30 - 5:20 Hodson 216
4 Luo Th 1:30 - 2:20 Bloomberg 278
5 Luo Th 3:00 - 3:50 Maryland 309

Your section number: __________________________

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.

- **No partial credit for any 1 point problem**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

- **Any answer or solution on scratch paper will receive no credit**.

Do not write in the table to the right.

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1. Determine which of the following transformations with domain \( P_2 \), the space of all polynomials of degree at most 2, is a linear transformation. Remember to justify your answers.

   (a) (1 point) \( T_1 : P_2 \to \mathbb{R}^4 \) defined by \( T_1(p) = (p(1), p(2), p(3), p(4)) \).

   (b) (1 point) \( T_2 : P_2 \to \mathbb{R}^3 \) defined by \( T_2(p) = (p(0), p'(0), p''(0)) \).
(c) (1 point) $T_3 : P_2 \to \mathbb{R}^3$ defined by $T_3(p) = (p(1) + 2, (p(0))^3, p'(0))$.

(d) (1 point) $T_4 : P_2 \to P_2$ defined by $T_4(p)(x) = xp'(x)$. 
2. (a) (3 points) Let $\mathbb{R}^{2 \times 2}$ to be the space of $2 \times 2$ matrices and consider the ordered basis $B$ of $\mathbb{R}^{2 \times 2}$,

$$B = \left( \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

For the linear transformation $T : \mathbb{R}^{2 \times 2} \to \mathbb{R}^{2 \times 2}$ defined by

$$T(A) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A - A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

determine $[T]_B$, the $B$-matrix of $T$. 
(b) (2 points) Find a basis of $\text{im}(T) \subset \mathbb{R}^{2 \times 2}$. 
3. (a) (3 points) Perform the Gram-Schmidt process to \( \mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^4 \), \( \mathbf{v}_1 = (1, 2, 3, 4) \), \( \mathbf{v}_2 = (4, 3, 2, 1) \), and find the corresponding QR factorization of the matrix

\[
M = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}
\]
(b) (3 points) Compute the $4 \times 4$ matrix (with respect to the standard basis) for the orthogonal projection onto the plane spanned by the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ above.
4. (a) (3 points) Can you find a $2019 \times 2019$ matrix $A$ such that $\text{im}(A^2) = \ker(A^2)$? If so, please give an example; otherwise please explain why.

(b) (1 point) Can you find a $4 \times 4$ matrix $A$ such that $\text{im}(A^2) = \ker(A^2)$? If so, please give an example; otherwise please explain why.
(c) (1 point) Let $P_{\geq 2}$ be the space of all polynomials of exactly degree 2. Prove or disprove: $P_{\geq 2}$ is a linear space.
Please estimate your score. You can also provide your suggestion for the course here.