

Homotopy Groups of Spheres

This table gives $\pi_r(S^n)$ for a range of values of r and n . In the table, k denotes the cyclic group \mathbf{Z}/k . The groups B, C, D, E, F denote respectively the direct sum of 2, 3, 4, 5, 6 copies of $\mathbf{Z}/2$.

See *Composition methods in homotopy groups of spheres* by H. Toda, Annals of Math. Studies 48 (Princeton Press, 1962) for this table and much more.

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$r = 1$	\mathbf{Z}	0	0	0	0	0	0	0	0	0
$r = 2$	0	\mathbf{Z}	0	0	0	0	0	0	0	0
$r = 3$	0	\mathbf{Z}	\mathbf{Z}	0	0	0	0	0	0	0
$r = 4$	0	2	2	\mathbf{Z}	0	0	0	0	0	0
$r = 5$	0	2	2	2	\mathbf{Z}	0	0	0	0	0
$r = 6$	0	12	12	2	2	\mathbf{Z}	0	0	0	0
$r = 7$	0	2	2	$\mathbf{Z}+12$	2	2	\mathbf{Z}	0	0	0
$r = 8$	0	2	2	2+2	24	2	2	\mathbf{Z}	0	0
$r = 9$	0	3	3	2+2	2	24	2	2	\mathbf{Z}	0
$r = 10$	0	15	15	24+3	2	0	24	2	2	2
$r = 11$	0	2	2	15	2	\mathbf{Z}	0	24	2	2
$r = 12$	0	2+2	2+2	2	30	2	0	0	24	24
$r = 13$	0	12+2	12+2	2+2+2	2	60	2	0	0	0
$r = 14$	0	84+B	84+B	120+12+2	C	24+2	120	2	0	0
$r = 15$	0	2+2	2+2	84+E	72+2	C	C	$\mathbf{Z}+120$	2	2
$r = 16$	0	6	6	F	504+B	72+2	D	D	240	240
$r = 17$	0	30	30	24+6+2	C	504+4	24+2	E	C	240
$r = 18$	0	30	30	2520+6+2	6+2	240	504+2	24+24+2	D	240
$r = 19$	0	6+2	6+2	30	6+2	6	0	504+2	24+2	$\mathbf{Z}+240$
$r = 20$	0	12+B	12+B	6+6+2	30+2	12+2	6	0	504+2	120
$r = 21$	0	12+B	12+B	24+12+4+B	2+2	60+6	24+4	6+2	0	504
$r = 22$	0	132+2	132+2	120+12+E	4+B	504+B	120+C	240+24+4	6	504
$r = 23$	0	?	?	132+E	24+B	D	D	120+E	16+4	504