

Basic Laplace Transforms

Reference: Simmons (Second Edition), Chapter 9

function	transform	comments
$f(x)$ defined for $x > 0$	$F(p) = L[f(x)]$ defined for $p > p_0$ (or $p \geq p_0$)	logically $(Lf)(p)$
$f(x) + g(x)$	$F(p) + G(p)$	L additive
$cf(x)$	$cF(p)$	c constant
1	$\frac{1}{p}$	
x	$\frac{1}{p^2}$	
x^2	$\frac{2}{p^3}$	
$\frac{1}{\sqrt{x}}$	$\frac{\sqrt{\pi}}{\sqrt{p}}$	
x^n	$\frac{n!}{p^{n+1}} = \frac{\Gamma(n+1)}{p^{n+1}}$	constant $n > -1$
e^{ax}	$\frac{1}{p-a}$	a constant
$\cos ax$	$\frac{p}{p^2 + a^2}$	a constant
$\sin ax$	$\frac{a}{p^2 + a^2}$	a constant
$\delta(x)$	1	Dirac delta “function”
$\delta(x-a)$	e^{-ap}	shift by constant a
$e^{ax}f(x)$	$F(p-a)$	a constant
$f'(x)$	$pF(p) - f(0+)$	
$f''(x)$	$p^2F(p) - pf(0+) - f'(0+)$	
$\int_0^x f(u)du$	$\frac{F(p)}{p}$	
$xf(x)$	$-F'(p)$	
$\frac{f(x)}{x}$	$\int_p^\infty F(v)dv$	
$(f * g)(x)$	$F(p)G(p)$	$\int_0^x f(x-u)g(u)du$