Second Examination

10:00 a.m. Edition

50 minutes. Closed book. No notes. No calculators.
80 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Make sure your T.A.'s name is on each book, as well as your name.
Do not evaluate square roots, trigonometric functions and such.
Use only the officially provided blue books.

1. The point $P = (1, 1, 2)$ lies on the surface S defined by the equation

$$x^3 + xy^2 + y^2 - z^2 + xyz = 1.$$

(a) Find the equation (in terms of x, y, and z) of the tangent plane to S at P.
(b) Find a unit normal vector to S at the point P.

2. (a) Find all the second order partial derivatives of the function $f(x, y) = x^2 e^{xy}$.
(b) Find the first order partial derivatives of the function $z = g(x, y)$ defined implicitly by the equation

$$6xy^2 + 3yz + z^3 = 0.$$

3. Evaluate the double integral

$$\int\int_\Omega x^2y^2dxdy$$

over the triangle Ω with vertices $(0, 0)$, $(1, 0)$, and $(1, 2)$.

4. A rectangular box with an open top is to be constructed out of sheet material. The material for the four sides costs $2 per square foot, while the material for the bottom of the box costs $3 per square foot. Find the dimensions of the least expensive box that has a volume of 6 cubic feet.