

## Final Examination

*50 minutes. Open book.*

*60 points, 20 per question.*

*Begin each question on a new page and number it clearly in the margin.*

1. Consider the projection  $\pi_1: X \times Y \rightarrow X$  to the first factor.
  - (a) If  $Y$  is compact, show that  $\pi_1$  is a *closed* map, i.e. that  $\pi_1(C)$  is closed whenever  $C \subset X \times Y$  is closed.
  - (b) What happens if  $Y$  is not compact?
2. Let  $A$  be a closed subspace of  $\mathbf{R}^2$ .
  - (a) If  $A$  is connected, is it necessarily true that the *interior* of  $A$  is connected?
  - (b) If  $A$  is connected, is it necessarily true that the *boundary* of  $A$  is connected?
  - (c) What about the converses of (a) and (b)?
3. What can you say about the fundamental groups of the following spaces:
  - (a) The space  $S^1 \times \mathbf{R}$  with one point deleted;
  - (b) The space  $\mathbf{R}^3$  with the line  $x = y = z$  deleted;
  - (c) The union of the three axes in  $\mathbf{R}^3$ ;
  - (d)  $\mathbf{R} \times \mathbf{Z}$ .