First Examination

10:00 a.m. Edition

50 minutes. Closed book. No notes. No calculators.
80 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Make sure your T.A.'s name is on each book, as well as your name.
Do not evaluate square roots, trigonometric functions and such.
Use only the officially provided blue books.

1. The line l is given parametrically by the equation

$$\mathbf{r}(t) = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + t(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}).$$

Let p be the plane that contains l and the point $(3, 2, 5)$.
(a) Find a normal vector to the plane p.
(b) Find the equation of the plane p in cartesian coordinates x, y, z.

2. The curve C is given parametrically by the equation

$$\mathbf{r}(t) = t^2\,\mathbf{i} + (t^3 - 2t + 2)\mathbf{j} + (t^4 - 2t + 1)\mathbf{k}.$$

Let P be the point on C where $t = 1$.
(a) Find the *unit* tangent vector to C at P.
(b) Find a parametric equation of the *tangent line* to C at the point P.

3. (a) Find the arc length along the curve

$$\mathbf{r}(t) = \frac{2t^3 + 5}{3}\,\mathbf{i} + (t^2 + 1)\mathbf{j} + (t + 1)\mathbf{k}$$

between the points $Q = (1, 2, 0)$ and $R = (7, 5, 3)$. [*Hint:* No difficult integrals are needed.]
(b) Find the direct straight-line distance between Q and R.

4. (a) Find the angle between the vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}$.
(b) What is the angle between the vectors $\mathbf{a}$ and $-2\mathbf{b}$?