Mid-Term Examination

50 minutes. Closed book. No notes. No calculators.
40 points, 10 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Use only the officially provided blue books.

1. Determine whether or not each of the following sets is countable. Give reasons.
(a) The set of all functions $\{0, 1, 2\} \to \mathbb{Z}_+$;
(b) The set of all subsets of $\mathbb{Z}_+$;
(c) The set of all functions $\{0, 1, 2\} \to \mathbb{R}$;
(d) The set of all functions $f\colon \mathbb{Z}_+ \to \mathbb{Z}_+$ that are *eventually constant*, i.e., there exists n_0 such that $f(n) = f(n_0)$ for all $n > n_0$.

2. For each of the following statements, either prove it is true or give a counterexample. If it is false, state whether it becomes true with $=$ replaced by $\subset$ or $\supset$. A, B, C and D are any sets.
(a) $A \times (B - C) = (A \times B) - (A \times C)$;
(b) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

3. Given a set X, define $d\colon X \times X \to \mathbb{R}$ by

$$d(x, y) = \begin{cases} 1 & \text{if } x \neq y; \\ 0 & \text{if } x = y. \end{cases}$$

(a) Prove that d is a metric on X.
(b) Describe the topology on X induced by d.

4. Define the relation $\sim$ on $\mathbb{R}_+$ (the positive real numbers) by $x \sim y$ if and only if x and y have the same integer part.
(a) Show that $\sim$ is an equivalence relation.
(b) Describe the equivalence classes of $\sim$.