Mid-Term Examination

by J. Michael Boardman

50 minutes. Open book. No notes.
40 points, 10 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Use only the officially provided blue books.
$\mathbb{Z}_+$ *denotes the set of positive integers.*

1. Determine whether each of the following sets is finite, countably infinite, or uncountable. Give reasons.
(a) The set of all finite subsets of $\mathbb{Q}$, the rationals;
(b) The set of all metrics on a finite set X;
(c) The set of all topologies on a finite set X;
(d) The set of all equivalence relations on the set $\mathbb{Z}_+$.

2. Give $\mathbb{Z}_+$ the *finite complement* topology, in which the open sets are the empty set $\emptyset$ and all sets of the form $\mathbb{Z}_+ - F$ where F is finite. Describe the open sets in the resulting product topology on $\mathbb{Z}_+ \times \mathbb{Z}_+$. Is this topology the same as the finite complement topology on $\mathbb{Z}_+ \times \mathbb{Z}_+$?

3. Let (X, d) be a metric space. Which of the following functions on $X \times X$ are metrics on X?
(a) d_1, defined by $d_1(x, y) = kd(x, y)$, where k is a positive real number;
(b) d_2, defined by $d_2(x, y) = d(x, y)^2$;
(c) d_3, defined by $d_3(x, y) = \sqrt{d(x, y)}$.

4. Let X be an ordered set, and Y a subset. Compare the *order* topology on Y, defined by treating Y as an ordered set with ordering induced from X, and the *subspace* topology, defined by treating Y as a subspace of X with its order topology, as follows:
(a) If the subset $U \subset Y$ is open in the order topology, is it necessarily open in the subspace topology?
(b) If $U \subset Y$ is open in the subspace topology, is it necessarily open in the order topology?

YOU MAY RETAIN THIS QUESTION SHEET