110.201 LINEAR ALGEBRA 5 November 1997

Second Examination

50 minutes. Closed book. No notes. 80 points, 20 per question.
Partial credit may be available, but only if you show your working.
HINT: There are many opportunities to check your answers.
Leave expressions such as $\sqrt{2}$ *and* $\cos^{-1}(1/3)$ *alone; do not evaluate them.*
No answer should contain an unexpanded determinant.
Begin each of the four questions on a new page and number it clearly in the margin.

1. A triangle in $\mathbf{R}^3$ has the vertices $A = (1, 0, 1)$, $B = (3, 2, 3)$ and $C = (2, 4, 3)$.
(a) Find the angle at A.
(b) Find (by any method) an equation for the plane that contains the triangle ABC.

2. A right-angled triangle in $\mathbf{R}^4$ has the vertices $P = (2, 5, 2, 2)$, $Q = (1, 1, 3, 3)$ and $R = (0, 3, 1, 2)$.
(a) At which vertex is the right angle? Explain how you know.
(b) Find the lengths of all three sides.
(c) Verify Pythagoras's Theorem for this triangle.

3. (a) Find the matrix of the linear transformation of the plane $\mathbf{R}^2$ that rotates every vector *clockwise* through an angle of 60 degrees. (Recall that $\cos(\pi/3) = 1/2$ and that $\sin(\pi/3) = \sqrt{3}/2$.)
(b) Find the matrix of the dilation of $\mathbf{R}^2$ that makes every vector three times as long (and leaves its direction unchanged).
(c) Find the matrix of the linear transformation of $\mathbf{R}^2$ that rotates every vector clockwise through an angle of 60 degrees *and* makes it three times as long.

4. Find the matrix of the orthogonal projection P in $\mathbf{R}^3$ to the plane $2x+2y+z = 0$. (You may assume P is linear.) Hint: An expression for $P(\mathbf{x})$ will be helpful.

THIS QUESTION SHEET MAY BE RETAINED JMB