Second Examination

11:00 a.m. Edition

50 minutes. Closed book. No notes. No calculators.
80 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Make sure your T.A.'s name is on each book, as well as your name.
Do not evaluate square roots, trigonometric functions and such.
Use only the officially provided blue books.

1. Find the first partial derivatives of the following functions,

(a) $f(x, y) = \ln(x + \ln(y))$, (b) $z = g(x, y)$,

where the function g is defined implicitly by the equation

$$x^2 + y^3 + 3xz + z^3 = 0.$$

2. Use differentials to approximate the value of $\sqrt{99}\sqrt[3]{124}$, given that $\sqrt{100} = 10$ and $\sqrt[3]{125} = 5$.

3. Suppose z is the function $h(x, y)$ of x and y defined by the equation

$$z = h(x, y) = 2x^2 + y^2 - 10.$$

(a) Find the direction in which the rate of change of z at the point $(2, 2)$ is maximal. What is this maximal rate of change?
(b) Find an equation of the tangent plane to the surface $z = h(x, y)$ at the point $(2, 2, 2)$.
(c) Find an equation of the normal line to the surface $z = h(x, y)$ at $(2, 2, 2)$.

4. Find the extreme values of the function $f(x, y) = x^2 - 4y^2$ on the region bounded by the ellipse $x^2 + 2y^2 = 1$, and the points where these values are attained.