110.202 CALCULUS III 7 October 1998

# First Examination

11:00 a.m. Edition

*50 minutes. Closed book. No notes.*
*80 points, 20 per question.*
*Partial credit may be available, but only if you show your working.*
*Begin each of the four questions on a new page and number it clearly in the margin.*
*Make sure your T.A.'s name is on each book, as well as your name.*
*Calculators are allowed but are not recommended or useful.*
*Do not numerically evaluate square roots, trigonometric functions and such.*

**1.** (a) Decompose the vector $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ as the sum $\mathbf{c} + \mathbf{d}$ of two vectors, where $\mathbf{c}$ is parallel to the vector $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{d}$ is perpendicular (orthogonal) to $\mathbf{b}$.

(b) Find the area of a triangle that has the vectors $\mathbf{a}$ and $\mathbf{b}$ as two of its sides.

**2.** A particle traces out the curve $C$. Its position at time $t$ is given by

$$\mathbf{r}(t) = t\,\mathbf{i} + (3/2)t^2\,\mathbf{j} + t^2\,\mathbf{k}.$$

(a) Find its velocity at any time $t$.

(b) Find the coordinates of every point where $C$ intersects the plane $2x+2y-z = 0$.

(c) Find the angle between the curve $C$ and the plane at *one* point of intersection (your choice) in (b).

**3.** The line $l$ is given parametrically by the equation

$$\mathbf{r}(t) = 2\mathbf{i} - 2\mathbf{j} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}).$$

The point $P_1$ has coordinates $(7, 2, 3)$.

(a) Find an equation for the plane that contains the line $l$ and the point $P_1$.

(b) Find the (shortest) distance between the point $P_1$ and the line $l$.

**4.** A spring $S$ is parametrized by the equation

$$\mathbf{r}(t) = t\cos t\,\mathbf{i} + t\sin t\,\mathbf{j} + 2t\,\mathbf{k}.$$

Let $P_2$ be the point of $S$ where $t = \pi$.

(a) Find the unit tangent vector to $S$ at $P_2$.

(b) Find a parametric equation for the tangent line to $S$ at $P_2$.

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