110.202 CALCULUS III 7 October 1998

First Examination

12:00 noon Edition

50 minutes. Closed book. No notes.
80 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each of the four questions on a new page and number it clearly in the margin.
Make sure your T.A.'s name is on each book, as well as your name.
Calculators are allowed but are not recommended or useful.
Do not numerically evaluate square roots, trigonometric functions and such.

5. Consider the line l parametrized by the equation

$$\mathbf{r}(t) = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(\mathbf{i} + \mathbf{j} - 2\mathbf{k}).$$

Let P_1 be the point $(3, 4, 5)$.
(a) Find an equation for the plane that contains P_1 and the line l.
(b) Find the (shortest) distance between the point P_1 and the line l.

6. The position at time t of a particle moving along a helix is given by

$$\mathbf{r}(t) = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j} + t^2\,\mathbf{k}.$$

(a) Find the velocity at any time $t > 0$.
(b) Find the speed at any time $t > 0$.
(c) Find the unit tangent vector to the helix at any time $t > 0$.
(d) Find the curvature of the helix at any time $t > 0$.

7. Consider the twisted cubic curve C parametrized by

$$\mathbf{r}(t) = (t^3/3)\mathbf{i} + t^2\,\mathbf{j} + 2t\,\mathbf{k}.$$

(a) Find a parametrization of the tangent line to C at the point where $t = 1$.
(b) Find the arc length along C between the points where $t = 0$ and $t = 2$.

8. (a) Decompose the vector $\mathbf{a} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ as the sum $\mathbf{c} + \mathbf{d}$ of two vectors, where $\mathbf{c}$ is parallel to the vector $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{d}$ is perpendicular (orthogonal) to $\mathbf{b}$.

(b) Find the area of a triangle that has the vectors $\mathbf{a}$ and $\mathbf{e}$ as two of its sides, where $\mathbf{e} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$.