Mid-Term Examination

50 minutes. Open book.
60 points, 20 per question.
Partial credit may be available, but only if you show your working.
Begin each question on a new page and number it clearly in the margin.

1. Given a metric space (X, d), define

$$d'(x, y) = \frac{d(x, y)}{1 + d(x, y)}$$

for all $x, y \in X$. Show that d' is a metric on X that defines the same topology as d.

2. Give $\mathbf{R}^2$ the euclidean metric, and the resulting standard topology. Which of the following collections of subsets are bases of the standard topology? In each case, give reasons.

(a) All open disks $B((x, y), r)$, where x, y, and r are required to be rational numbers whose denominators are powers of 2.

(b) All open squares $(a, b) \times (c, d)$, where a, b, c, and d are required to be integers.

(c) All open disks $B((x, y), r)$, where we require $r < 1/100$.

(d) All open disks $B((a/n, b/n), 1/n^4)$, where a, b and n are integers ($n > 0$).

3. Let A and B be any subsets of a topological space X.

(a) Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$?

(b) Is $\overline{A - B} = \overline{A} - \overline{B}$?

In each case, give either a proof or a counterexample.