1. Consider the second order differential equation
   \[ xu''(x) + 2u'(x) + (\lambda - x)u(x) = 0, \quad 1 < x < 2, \quad u(1) = u(2) = 0. \]
   a. (10pts) Put the equation in Sturm-Liouville form.
   b. (10pts) Write out the orthogonality condition for the eigenfunctions.
   c. (10pts) Show that all eigenvalues \( \lambda > 0 \).

2. Consider the boundary value problem
   \[ y''(x) + 4y(x) = f(x), \quad 0 < x < \pi, \quad y(0) = y(\pi) = 0. \]
   a. (15pts) Show that a necessary condition for a solution is that
      \[ \int_0^\pi f(x) \sin 2x \, dx = 0. \]
   b. (15pts) Assuming this condition, find the solution by the method of eigenfunction expansion.

3a (20pts). Find the Green’s function for the problem
   \[ (xu')' = f(x), \quad 1 < x < e, \quad u(1) = 0, \quad u(e) = 0, \]
   by direct construction from two linearly independent solutions. Hint: Find \( u_1(x), \ u_2(x) \) linearly solutions of \( (xu')' = 0 \) with \( u_1(1) = u_2(e) = 0 \)

3b. (20pts) Use the Green’s function to find the explicit solution for \( f(x) = x \). You do not need to derive the formula for the representation of the solution in terms of the Green’s function. The integration by parts formula, \( \int (y \log y \, dy = \frac{1}{2} y^2 \log y - \frac{1}{4} y^2 \) will be handy. Check directly that your solution is correct.