Problem Set 5 (due Thursday Oct. 15)

You may discuss the problems with your classmates but you must write your own solutions. Be clear and to the point. Page numbers refer to the text “Variational methods in Optimization” by Donald R. Smith.

126:4,5,6,7,8

6. Find the extremals of $I(y) = \int_0^1 (y'^2 + x^2)dx, y(0) = y(1) = 0$ subject to $\int_0^1 y^2 dx = 2$.

7. Find the extremals for $I(y) = \int_0^\pi (y'^2 + 2 \sin xy)dx, y(0) = y(\pi) = 0$ subject to $\int_0^\pi y(x)dx = 1$.

8. Find the extremals of

$$J(y, z) = \int_0^{\pi/2} (y'^2 + z'^2 + 2yz) \, dx \quad y(0) = z(0) = 0, \; y(\pi/2) = z(\pi/2) = 1.$$