Problem Set 1 Due Wednesday February 10 in class

You may discuss problems with classmates but write your own solutions. Write carefully and rigorously and to the point, preferably using LaTeX.

Read sections 3.1, 3.2.

1. Evans p162: 2a,b
2. Show that the general solution of $u_x + xu_y = f(x,y)$ is given by

$$u(x,y) = \phi(y - \frac{x^2}{2}) + \int_0^x f(\tau, \frac{t^2}{2} + y - \frac{x^2}{2}) d\tau \ .$$

3. Evans p.162: 3a,b,c
4. Show that the equation

$$a\phi_x + (bx - 1 + c\phi)\phi_y = d, \quad \phi(x,0) = 0, \tag{1}$$

where a,b,c,d are given constants, has a real analytic solution

$$\phi(x,y) = \frac{d}{cd-ab}\big(1 - bx - \sqrt{(1-bx)^2 + 2(cd-ab)y}\,\big) \quad \text{if } cd - ab \neq 0 \ . \tag{2}$$

Also find the explicit solution if $cd = ab$.
The solution (2) will be used in the proof of the Cauchy-Kowaleski theorem.