Problem 1 Find the limit, if it exists, or show that the limit does not exist.

1. (10pts) \[ \lim_{(x,y) \to (\pi,2)} \sin^3(x)e^{-xy^2+\cos(y^2)} \].

2. (10pts) \[ \lim_{(x,y) \to (0,0)} \frac{xy}{(\sqrt{x+1}-1)(\sqrt{y+1}-1)} \].

3. (20pts) \[ \lim_{(x,y) \to (1,1)} \frac{(x-y)^2}{(x-1)(y-1)} \].

Problem 2 Find the indicated partial derivatives.

1. (10pts) \( f(x, y) = 3x - 2y^4 \), \( f_x \) and \( f_y \).

2. (10pts) \( f(x, y) = \ln \sqrt{x^2 + y^2} \), \( f_{xy} \).

Problem 3 (20pts) Let \( A \) be a \( 2 \times 2 \) matrix defined by

\[
\begin{pmatrix}
2 & 1 \\
2 & 3
\end{pmatrix}
\]

Find its eigenvalues \( \lambda_1 \) and \( \lambda_2 \) and corresponding eigenvectors.

Problem 4 (20pts) Let \( f(x, y) \) be a function defined on \( \mathbb{R}^2 \) by \( f(x, y) = \frac{xy}{x^2+y^2} \) for all \((x, y) \neq (0, 0) \) and \( f(0,0) = 0 \).

1. Find \( f_x(0, 0) \) and \( f_y(0,0) \).

2. Are \( f_x \) and \( f_y \) continuous on \( \mathbb{R}^2 \)?