Problem 1 Solve the following linear equations.

1. \[\begin{align*}
2x + 5y - z &= 0 \\
-x + y + 2z &= 0 \\
-2x + 3y + 2z &= 1
\end{align*}\]

Answer: \[x = \begin{bmatrix}
\dfrac{11}{17} \\
\dfrac{3}{17} \\
\dfrac{-7}{17}
\end{bmatrix}^T.
\]

2. \[
\begin{pmatrix}
2 & 3 & -1 \\
0 & 2 & 1 \\
1 & 0 & -2
\end{pmatrix} x = \begin{pmatrix}
0 \\
1 \\
2
\end{pmatrix}.
\]

Answer: \[x = \begin{bmatrix}
\dfrac{-16}{3} \\
\dfrac{7}{3} \\
\dfrac{-11}{3}
\end{bmatrix}^T.
\]

Problem 2 Find the indicated partial derivatives.

1. \[f(x, y) = \ln(x^2 + y^2), \quad f_{xy}.
\]
   Answer: \[f_{xy} = \dfrac{-4xy}{(x^2 + y^2)^2}.
\]

2. \[u = x^a y^b z^c, \quad \frac{\partial^6 u}{\partial x^2 \partial y^3 \partial z}.
\]
   Answer: \[\frac{\partial^6 u}{\partial x^2 \partial y^3 \partial z} = abc(b-1)(c-1)(c-2)x^{a-1}y^{b-2}z^{c-3}.
\]

Problem 3 Find all local maximum and minimum values and saddle points of each given function.

1. \[f(x, y) = e^x \cos y.
\]

2. \[f(x, y) = x^3 y + 12x^2 - 8y.
\]

3. \[f(x, y) = e^{4y-x^2-y^2}.
\]

Problem 4 Solve the given differential equations.

1. \[\frac{dx}{dt} = tx.
\]

2. \[
\frac{dx}{dt} = \begin{pmatrix}
2 & 8 \\
1 & 4
\end{pmatrix} x.
\]
3. \[
\frac{dx}{dt} = \begin{pmatrix} -3 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}, \]
\[
\mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.
\]

4. \[y''(t) + y'(t) - 1 = 0.\]

**Problem 5** Consider differential equations of the form
\[
\frac{dx}{dt} = A \mathbf{x}(t).
\]
Classify the equilibrium \((0,0)\) and analyze the stability.

1. \[
A = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}.
\]

2. \[
A = \begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}.
\]

**Problem 6** Let \(S\) be the unit sphere in \(\mathbb{R}^3\) defined as \(S = \{ (x, y, z) \mid x^2 + y^2 + z^2 = 1 \}\). Find a point \(A = (x, y, z) \in S\) that has the smallest distance from \((3, 4, 5)\).