Questions for Midterm III review from Final Fall 06

3. (a) Let $A = \begin{bmatrix} 4 & -3 \\ -3 & -4 \end{bmatrix}$. Determine a diagonal matrix D, and an orthogonal matrix S for which $A = SDS^{-1}$. Multiply out SDS^{-1} to check that your answer is correct.
(b) Let $B = \begin{bmatrix} 4 & 3 \\ -3 & -4 \end{bmatrix}$. Determine whether B is similar to the matrix A from part (a) in $\mathbb{R}^{2\times 2}$.

5. (a) Let V be a linear space. Suppose that λ is an eigenvalue of the linear transformation $T : V \to V$. Derive the fact that λ^2 is an eigenvalue of T^2.
(b) Determine all matrices in $\mathbb{R}^{3\times 3}$ that are both symmetric and orthogonal, and describe them geometrically. [Suggestion: Express the two conditions in terms of 'transpose'.]

7. For which $a \in \mathbb{R}$, $b \in \mathbb{R}$ does the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ b & 1 & 0 \\ 0 & a & 1 \end{bmatrix}$ have an eigenbasis (for $\mathbb{R}^3$)?
When it does, specify an eigenbasis (depending on a and b).

8. Let V be $\text{Span}\{1, \sin x, \cos x\}$. The dimension of V is 3.
(c) Let D denote the linear operator on V given by $D(f) = f'$. Determine the complex eigenvalues of D-that includes the real ones!-and the corresponding eigenspaces.

9. Let $A = \begin{bmatrix} 1 & b \\ c & 1 \end{bmatrix}$, where b and c are real scalars. Determine the set of values of b and c for which the dynamical system $\mathbf{x}(t+1) = A\mathbf{x}(t)$ is asymptotically stable
(meaning: for all initial states, the state vector tends to $\mathbf{0}$, as $t \to \infty$.)

10. Determine whether $q(x_1, x_2) = x_1^2 + 3x_1x_2 + 2x_2^2 = 1$ is the equation of an ellipse.

11. (a) Give an example of a 2×2 real matrices that have the same characteristic polynomial yet they are not similar. Explain.
(b) True or False: If a matrix fails to diagonalize over $\mathbb{R}$, it will diagonalize over $\mathbb{C}$. Explain.