Questions for Midterm III Review from Final Spring 06

11. Find all eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}$.

12. For each of the eigenvalues of A, find the associated eigenspace.

13. Is it possible to diagonalize the matrix A?

19. True or False: If A and B are both symmetric matrices, them their product AB must also be symmetric. Explain the reasoning behind your answer.

20. True or False: If A and B are both orthogonal matrices, them their product AB must also be orthogonal. Explain the reasoning behind your answer.

21. How many complex eigenvalues does the matrix $M = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 2 & 0 & 3 & 3 \\ 1 & 2 & 3 & 0 & 4 \\ 1 & 2 & 3 & 4 & 0 \end{bmatrix}$ have?

22. Express the quadratic form $q(x_1, x_2) = x_1^2 + 6x_1x_2 + 8x_2^2$ as an inner product $q(\mathbf{x}) = \langle \mathbf{x}, A\mathbf{x} \rangle$, where A is a symmetric matrix.

23. Is there a choice of numbers (x_1, x_2) for which $q(x_1, x_2)$ is negative? What does the set of points where $q(x_1, x_2) = 1$ look like? [Please describe the overall shape of the set - it is not necessary to give exact specifications.]

24. What are the singular values of matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$?

25. Find a set of perpendicular vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ in $\mathbb{R}^2$ which have the additional property that $A\mathbf{v}_1$ and $A\mathbf{v}_2$ are also perpendicular to each other?

Remark The vectors can be used to obtain the singular value decomposition $A = U\Sigma V^T$, where $V = \begin{bmatrix} | & | \\ \mathbf{v}_1 & \mathbf{v}_2 \\ | & | \end{bmatrix}$, $\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \end{bmatrix}$, where σ_1 and σ_2 are the singular values, and $U = \begin{bmatrix} | & | & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \\ | & | & | \end{bmatrix}$, where $\mathbf{u}_1 = A\mathbf{v}_1/\sigma_1$, $\mathbf{u}_2 = A\mathbf{v}_2/\sigma_2$ and $\mathbf{u}_3$ is orthogonal to $\mathbf{u}_1$ and $\mathbf{u}_2$ and normalized.