Questions for Midterm III Review from Final Spring 09

1. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

(a) Find the eigenvalues of A.
(b) Is A diagonalizable? explain why or why not?

2. Let A be a 2×2 matrix with eigenvalues $1/2$ and $-1/2$.
Let $\text{Ker}\left(A - \frac{1}{2}I\right) = \text{Span}\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}\}$ and $\text{Ker}\left(A + \frac{1}{2}I\right) = \text{Span}\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$.
(a) Let $\mathbf{x}(t+1) = \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} = A\mathbf{x}(t)$. Given that $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find $\mathbf{x}(3)$.
(b) Draw the phase portrait for the discrete system in part (a).

3. Let $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

(a) Given that $\lambda = 1$ and 5 are the only eigenvalues of A. Find an orthonormal basis of $\mathbb{R}^3$ denoted by $\mathcal{B}$ consisting of eigenvectors of A.
(b) Given the following quadratic form $q(x_1, x_2) = 3x_1^2 + 4x_1x_2 + 3x_2^2$.
Describe q in terms of $\mathcal{B}$ coordinates. Show work.

8. Let A be a 2×2 matrix with eigenvalues 1 and 3,
such that $\text{Ker}\left(A - I\right) = \text{Span}\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}\}$ and $\text{Ker}\left(A - 3I\right) = \text{Span}\{\begin{bmatrix} 1 \\ -3 \end{bmatrix}\}$.
(a) Find A. Show work.
(b) Let T denote the transformation $T\mathbf{x} = A\mathbf{x}$. Write down the matrix of the transformation T with respect to the basis $\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \end{bmatrix}\}$. Show work.

10. State true or false with justification.
10(i) If A is a orthogonal 3×3 matrix then $\det A > 0$.
10(iv) If A is a 2×2 symmetric matrix then all its eigenvalues are positive real numbers.