NAME:

SECTION # OR TA'S NAME:

LINEAR ALGEBRA – FIRST MIDTERM EXAM – OCTOBER 17, 2001

Please attempt all the problems and show all your work. Don't hesitate to ask me for clarification on any questions you may have. You may **not** *use any notes, books or calculators.*

1 . Solve the following system of linear equations:

$$\begin{aligned} x + y - z &= 1 \\ -5x + y + z &= -7 \\ x - 5y + 3z &= 3 \end{aligned}$$

Is the solution unique?

2 . Find a basis for the kernel and a basis for the image of the following matrix:

$$\begin{pmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & -2 & 2 \\ 1 & -1 & -2 & 0 & 3 \\ 2 & -2 & -1 & 3 & 4 \end{pmatrix}$$

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3 . Answer the following questions **true or false**:

(i) The rank of the following matrix is three: $\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$?

(ii) If $\vec{u}, \vec{v}, \vec{w}$ are vectors in $\mathbb{R}^n$, and $\vec{u}$ is a linear combination of $\vec{v}$ and $\vec{w}$, then $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

(iii) If A is an invertible $n \times n$ matrix, and B is any $n \times m$ matrix, then $\ker(AB) = \ker(B)$.

(iv) If $T : \mathbb{R}^m \to \mathbb{R}^n$ is linear, and $\vec{b} \in \mathbb{R}^n$, then the set of solutions to the equation $T\vec{x} = \vec{b}$ is a linear subspace of $\mathbb{R}^m$.

4 . Consider the vectors $\vec{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{u}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given by:

$$T\vec{x} = (\vec{x} \cdot \vec{u}_1)\, \vec{u}_2 + (\vec{x} \cdot \vec{u}_2)\, \vec{u}_1 \ .$$

Show that T is a linear transformation, and compute its matrix (with respect to the standard basis).

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5 . Let $\mathcal{B} = \{\vec{v}_1, \vec{v}_2\}$ be a basis for $\mathbb{R}^2$, where $\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Suppose that the matrix of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with respect to the standard basis $\mathcal{E}$ of $\mathbb{R}^2$ is

$$[T]_{\mathcal{E}} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix} .$$

Calculate $[T]_{\mathcal{B}}$, the expression of T with respect to the basis $\mathcal{B}$.

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