Name: ______________________________ **Section Number:** ____________

110.201 Linear Algebra
FALL 2013
MIDTERM EXAMINATION
October 11, 2013

Instructions: The exam is **7** pages long, including this title page. The number of points each problem is worth is listed after the problem number. The exam totals to one hundred points. For each item, please **show your work** or **explain** how you reached your solution. Please do all the work you wish graded on the exam. Good luck !!

PLEASE DO NOT WRITE ON THIS TABLE !!

Problem	Score	Points for the Problem
1		30
2		30
3		15
4		25
TOTAL		100

Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________________ **Date:** ____________

Question 1. [30 points] For the system $x - y + 3z = 1$, $y = -2x + 5$, $9z - x - 5y + 7 = 0$, do the following:

(a) Write the system in the matrix form $\mathbf{Ax} = \mathbf{b}$, for $\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

(b) Write out the augmented matrix for this system and calculate its row-reduced echelon form.

(c) Write out the complete set of solutions (if they exist) in vector form using parameters if needed.

(d) Calculate the inverse of the coefficient matrix $\mathbf{A}$ you found in part **(a)**, if it exists, or show that $\mathbf{A}^{-1}$ doesn't exist.

Question 2. [30 points] Let V be the subspace of $\mathbb{R}^4$ given by all solutions to the equation $2x_1 - x_2 + 3x_3 = 0$.

(a) What is the dimension of V?

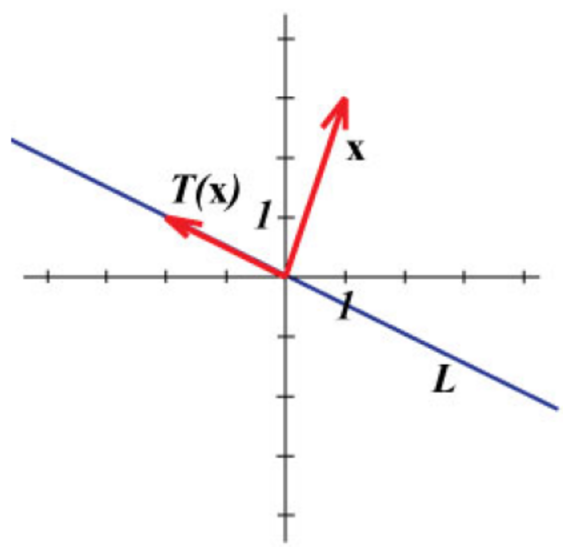
(b) Construct a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$, $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$, where $V = \operatorname{im}(\mathbf{A})$. Then use $\mathbf{A}$ to construct a basis for $\operatorname{im}(\mathbf{A})$. You will need to verify that what you have is a basis.

(c) Construct a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}$, $T(\mathbf{x}) = \mathbf{B}\mathbf{x}$, where $V = \ker(\mathbf{B})$. Then use $\mathbf{B}$ to construct a basis for $\ker(\mathbf{B})$. You will need to verify that what you have is a basis.

Question 3. [15 points] For $\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, let $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(\mathbf{x}) = \begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}\mathbf{x}$ be a linear transformation. Do the following:

(a) Write this transformation as a composition of a scaling and an orthogonal projection.

(b) Find the equation of the line $L = \operatorname{im}(T)$ and carefully draw L, $\mathbf{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$ on the graph provided.



Question 4. [25 points] Suppose we know for a linear transformation T of $\mathbb{R}^2$ that $T\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\5\end{bmatrix}$ and $T\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$. Do the following:

(a) Find the matrix $\mathbf{A}$ so that $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$.

(b) Given the basis $\mathcal{B} = \quad \begin{matrix}1\\-2\end{matrix} \;, \; \begin{matrix}3\\3\end{matrix} \quad$, find the matrix $\mathbf{B}$ so that $T\,[\mathbf{x}]_{\mathcal{B}} = B\,[\mathbf{x}]_{\mathcal{B}}$.

(c) Find the $\mathcal{B}$-coordinates of the vector $\mathbf{x} = \begin{bmatrix}2\\5\end{bmatrix}$.