

Johns Hopkins University
Math 201, Spring 2007
Name:
Section:

Midterm Exam # 1
Time: 50 minutes

No books, notes, calculators. Please explain carefully all steps leading to your solutions, or risk losing credit.

Problem 1: (4 points=2+2)

1. Let $r : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the rotation of angle θ around the origin. Write the matrix corresponding to r . Justify it by a drawing.
2. Let $R : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the rotation of angle θ around the line spanned by $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Write the matrix corresponding to R . Justify it by a drawing.

Problem 2: (8 points=4+2+1+1)

Consider the matrix:

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 2 & 3 & 2 \end{pmatrix}$$

1. Is A invertible? If yes, compute its inverse and check your answer by evaluating the matrix product AA^{-1} .
2. Solve the linear system $AX = Y$ for $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $Y = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.
3. What is $\text{rref}(A)$?
4. What is the rank of A ?

Problem 3: (8 points=1+4+3)

Consider the matrix:

$$B = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$$

1. What are the domain and codomain of the linear transformation f defined by B ?
2. Find a basis of the image $\text{Im}(B)$. What is the rank of B ?
3. Find a basis of the kernel $\text{Ker}(B)$. What is its dimension? How could you predict this value?