February 25, 2009

Name ....
Section/ Name of your TA ....

SUGGESTED SOLUTIONS TO MIDTERM EXAM 1 *100pts.*
MATH 201 VER ***

- **There are 6 pages in the exam including this page.**
- **Write all your answers clearly. You have to show work to get points for your answers.**
- **Read all the questions carefully and make sure you answer all the parts.**
- **You can write on both sides of the paper. Indicate that the answer follows on the back of the page.**
- **Use of Calculators is *not* allowed during the exam.**

(1) ..../20

(2) ..../22

(3) ..../22

(4) ..../36

Total ....../100

(1) *20pts.* Solve the following system of equations.

$$\begin{aligned} x_1 + 2x_3 &= 1 \\ x_1 + 2x_2 + 2x_3 &= 0 \\ x_1 + x_3 &= 2 \end{aligned}$$

(2) *22 pts.* Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined as

$$T\left(\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}\right) = \begin{bmatrix} y_1 + y_2 \\ y_1 - y_2 \\ y_3 \end{bmatrix}$$

(a) Find the Kernel of T.

(b) Is T invertible? Explain why or why not?

(3) *22 pts.* Is $W = \{ \begin{bmatrix} a-b \\ b \\ 0 \end{bmatrix} : a, b \in I\!R\}$ a subspace of $I\!R^3$ (This is the set of vectors in $I\!R^3$ of the form $\begin{bmatrix} a-b \\ b \\ 0 \end{bmatrix}$ for all possible real values of a and b) ?

(a) Show that W is a subspace of $I\!R^3$.

(b) Find a basis for W.

(4) *36 pts.* State whether the following statements are true or false. If true, explain your answer. If false, give an example for which the statement is false. Only TWO points for stating true or false. Each of this problem is worth 12 points.

(a) Let W be a subspace of $\mathbb{R}^4$. If $W = \text{Span}\{\vec{w}_1, \cdots, \vec{w}_k\}$ for vectors $\vec{w}_1, \cdots, \vec{w}_k$ in $\mathbb{R}^4$ and the dimension of W is 3 then $k = 3$.

(b) Let A and B be 2×2 matrices. Then $(A + B)^{-1} = A^{-1} + B^{-1}$.

(c) Let A be a 2×3 matrix. If the Rank $A = 2$ then the equation $A\vec{x} = \vec{0}$ has a unique solution.